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THE PROPAGATION OF LIGHT IN ROTATING SYSTEMS*

BY
L. SILBERSTEIN

The purpose of the present paper† is to investigate some questions concerning light propagation in a uniformly rotating rigid system, such as the Earth, on both the aether theory and the relativity theory.

On both theories we shall have to understand by "rotation" a rotation of our rigid system[1] S with uniform angular velocity *relatively to the fixed stars*, or to any other inertial system, which will be shortly referred to as the reference system S*. That such a specification of rotation is still necessary even in the relativity theory, in spite of appearances to the contrary, will become clear in the sequel where we shall also have the opportunity to point out some outstanding difficulties of the relativistic gravitation theory with respect to the concept of rotation.

With regard to the rotating system S itself, it will perhaps be well to have in mind our own Earth. The more so as the most interesting experiment in connection with our subject will be a purely terrestrial one.

1. To begin with the aether-theory treatment, let $\bar{\omega}$ be the (scalar) angular velocity of the Earth (S) relatively to the fixed

* Communication No. 123 from the Research Laboratory of the Eastman Kodak Company.

† Paper read December 29, 1920, at the Chicago Meeting of the Optical and the Physical Societies.

[1] It is well known that among the possible motions of a relativistically rigid body (as defined by Born and Herglotz) there is uniform rotation, such as is familiar to us from the kinematics of ordinary classical rigid bodies.

stars (S*) and let κ-1 be the rotatory dragging coefficient at and near the surface of the Earth, in other words, let $\kappa\tilde{\omega}$ be the relative angular velocity of the aether and the Earth. If the unit vector $\mathbf{k}$ is taken along the positive axis of rotation, the vector velocity of the aether stream past a point P of S will be

$$\mathbf{u}=\kappa\tilde{\omega}\mathbf{V}\mathbf{r}\mathbf{k},$$

$\mathbf{r}$ being the vector drawn from the center of the Earth (or from any point fixed on the axis) to the point P; in Cartesians, with $\mathbf{r}=x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$,

$$u_x=\kappa\tilde{\omega}y, \quad u_y=-\kappa\tilde{\omega}x, \quad u_z=0,$$

and the resultant velocity, $u=\kappa\tilde{\omega}r$, where r is distance from the axis. As to the factor κ, it is not our intention to prejudice its value, which may be any fraction from zero to unity, corresponding to a full drag and to no drag, respectively.

The propagation of light in S* being isotropic, of constant velocity c, the velocity of light in S, always in vacuo, along the wave-normal $\mathbf{n}$ (unit vector) will be

$$v=c+\mathbf{un}=c\left[1+\frac{\kappa\tilde{\omega}r}{c}\cos(\mathbf{u},\mathbf{n})\right] \quad \text{.....(1a)}$$

Notice that, in the case of the Earth, $\tilde{\omega}=2\pi/86164$, the reciprocal length $\tilde{\omega}/c$ amounts only to about 2.43×10^{-5}cm.$^{-1}$, so that even if r be of the order of the Earth's radius, the factor $\frac{\kappa\tilde{\omega}r}{c}$ is a very small fraction. Such being the case it will be enough to retain in all our formulae $\kappa\tilde{\omega}r/c$ itself, rejecting its square and higher powers. Now, rigorously speaking, formula (1) is valid for the *wave-normal* and not for the ray or the tangent to the "light path." But $\mathbf{n}$ appears in (1a) only in the term multiplied by $\kappa\tilde{\omega}r/c$, and since $\mathbf{n}$ differs from the light ray only by small terms, we have up to higher order terms, simply

$$\frac{v}{c}=1+\frac{1}{c}(\mathbf{up})=1+\frac{\kappa\tilde{\omega}r}{c}\cos\gamma, \quad \text{.....(1)}$$

where $\mathbf{p}$ is a unit vector along the optical ray, and γ the angle between $\mathbf{p}$ and $\mathbf{u}$.

Such being the expression for the velocity of light along the ray, we can at once find the shape of the ray or light path in S

by means of Fermat's law, which it will be enough to work out in detail for the case of a light ray contained in a plane parallel to the equatorial plane.[2] In fact let $d\sigma$ be a line-element of the ray; then Fermat's principle is $\delta \int \frac{d\sigma}{v} = 0$, the limits of the integral being fixed. Now, introducing in the said plane polar co-ordinates r, θ, the latter measured positively in the sense of the rotation of S, we have

$$\cos\gamma = -r\, d\theta/d\sigma = r\,\theta' \sin\gamma,$$

so that, by (1),

$$\frac{v}{c} = 1 + \frac{r^2\tilde{\omega}\theta'}{c} \sin\gamma,$$

and Fermat's principle becomes, after easy reductions, and considering as corresponding points those having the same r,

$$\int \left\{ \frac{\partial}{\partial\theta'} \left[\frac{c}{v\sin\gamma} \right] \right\} \delta\theta' .\, dr = 0.$$

Thus, $\frac{d}{dr}\left\{\right\} = 0$, and the required equation of the light path becomes

$$\frac{\partial}{\partial\theta'} \left[\frac{c}{v\sin\gamma} \right] = \text{const.}$$

Now $\frac{c}{v\sin\gamma} = -\frac{1+r^2\theta'^2}{\sqrt{1+r^2\theta'^2} - \frac{\kappa\tilde{\omega}r^2}{c}\theta'}$. Thus after simple reductions and putting $\gamma = \eta + \frac{\pi}{2}$,

$$\sin\eta + \frac{r\omega(1-2\sin^2\eta)}{(1-\omega r\sin\eta)^2} = \frac{A}{r}, \text{ where } \omega = \frac{\kappa\tilde{\omega}}{c},\ A = \text{const.}$$

Rejecting second order terms, the left hand member of this equation can be written $\sin\eta + \omega r$.

Ultimately, therefore, the equation of the light path in S becomes

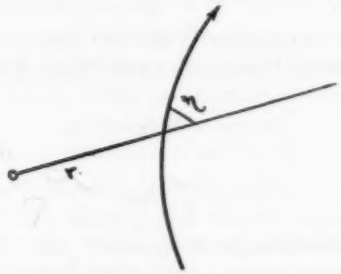
$$r\left(\sin\eta + \frac{r\kappa\tilde{\omega}}{c} \right) = A, \quad \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

---

[2] If the end-points (any two points) of a light path be in such a plane, the whole light path is contained in that plane.

where η is the angle under which the light path cuts the radius vector (Fig. 1).

Fig. 1



Notice that in absence of rotation (2) reduces, as it should, to $r \sin \eta = \text{const}$, this being the equation of a straight line. There is thus, due to rotation, a first-order deviation of the light rays from straight lines.

In order to introduce into the ray equation the polar coordinates r, θ instead of r, η, it is enough to remember that

$$\sin\eta = r\frac{d\theta}{dr}\sqrt{1+r^2\left(\frac{d\theta}{dr}\right)^2}.$$

Thus (2) becomes, with $\frac{1}{r}=\rho$, $B=1+2A\omega$, $\omega=\kappa\tilde{\omega}/c$,

$$-d\theta = \frac{\omega r dr}{\sqrt{Br^2-A^2}} + \frac{A d\rho}{\sqrt{B-A^2\rho^2}},$$

whence, integrating, putting $A/\sqrt{B}=r_o$, and counting $\theta-\theta_o$ from the radius vector $r=r_o$ (in the sense of rotation of S),

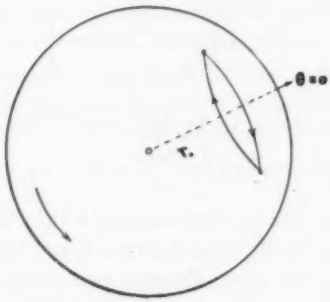
$$\cos\left[\theta-\theta_o \pm \omega\sqrt{\frac{r^2-r_o^2}{B}}\right]=\frac{r_o}{r}.$$

Here $B=1+2\omega A$ and $A=r_o(1+r_o\omega)$, so that rejecting second order terms and with the same right also replacing $\sqrt{r^2-r_o^2}$ by $r_o\tan\theta$, we have ultimately

$$\frac{r_o}{r}=\cos\left[\theta-\theta_o\pm\frac{\kappa\tilde{\omega}}{c}\sqrt{r^2-r_o^2}\right]=\cos\left[\theta-\theta_o\pm\frac{r_o\kappa\tilde{\omega}}{c}\tan(\theta-\theta_o)\right], \quad (3)$$

the upper or the lower sign to be taken for light travelling in the sense of rotation of S or in the opposite sense. Or, to put the sign

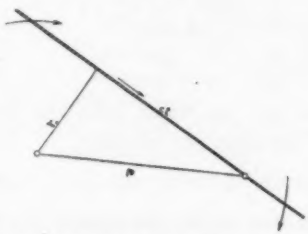
Fig. 2



rule in a more convenient form, the light path is convex (or bulges out) always to the left of a person[3] walking in the direction of propagation (Fig. 2).

This equation of the light ray can also be obtained more directly by transforming the light rays of S^* to our system S by means of the substitution $r = r'$, $\theta' = \theta + \kappa\tilde{\omega}t$ (and $t' = t$). In fact, the equa-

Fig. 3



tion of a straight line, and such is in S^* every optical ray, can be written

$$r' \cos\theta' = r_0' = \text{const.},$$

and this becomes, through the said transformation,

$$\frac{r_0}{r} = \cos(\theta + \kappa\tilde{\omega}t).$$

[3] For whom the rotation of S is clockwise.

In other words the light path is the path of a particle moving with uniform velocity c along a straight line which is itself spinning uniformly around O (Fig. 3). If $t=0$ for $r=r_0$ we have $ct=ct'=\sqrt{r^2-r_0^2}$, which gives again

$$\frac{r_0}{r}=\cos\left[\theta-\theta_0\pm\frac{\kappa\tilde{\omega}}{c}\sqrt{r^2-r_0^2}\right],$$

as under (3).

Developing the cosine, up to the second order, we can write

$$\frac{r_0}{r}=\cos(\theta-\theta_0).\left[1\mp\frac{\kappa\tilde{\omega}r_0}{c}\tan^2(\theta-\theta_0)\right]\ldots(3a)$$

The constants θ_0, r_0 can be determined if either the initial direction or any two points, A, B of the light path, say the sending and the receiving stations, are given through two pairs of r, θ. It will be noted that the light path or ray BA (i.e., with B as sending and A as receiving station) does not coincide with AB. The two optical rays AB and BA enclose between them a certain area, having the shape of a biconvex lens. In other words light propagation in S is irreversible. Under appropriate circumstances A will see B without being seen by B.

The last ray-equation can still be simplified. Putting the x-axis along the radius vector $r_0(\theta=\theta_0)$, so that

$$x=r\cos(\theta-\theta_0),\quad y=r\sin(\theta-\theta_0),$$

we have $x(1\mp\epsilon y^2/x^2)=r_0$, whence

$$x=r_0\left(1\pm\frac{\epsilon y^2}{x^2}\right),\qquad \epsilon=\frac{r_0\kappa\tilde{\omega}}{c}\ldots\ldots\ldots\ldots\ldots(4)$$

In the small term we can write, with the same approximation $x=r_0$, so that the ray equation becomes

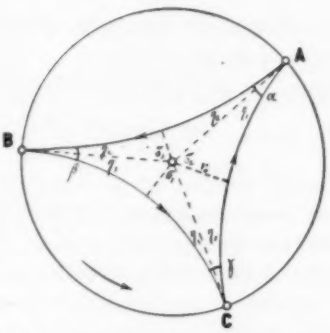
$$x=r_0\pm\frac{\kappa\tilde{\omega}}{c}y^2\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4a)$$

Thus, up to the second order, the optical ray (which more rigorously is a complicated spiral) becomes simply a parabola, with apex in shortest distance (r_0) from O. If the propagation is in the sense of rotation of S, the parabolic ray turns its convex side towards the axis of rotation. In the opposite case it will turn its concave side towards O. Various problems of what may be called optical trigonometry of the rotating system, i.e., problems concerning triangles or polygons built up of optical rays, can

now be easily dealt with, namely by constructing the said parabolic arcs as light paths and keeping well in mind the sense of propagation.

By way of illustration consider an optical triangle traversed, say, in the sense of rotation, and having its corners A, B, C on the circle $r=a$. Given the angles $2\sigma_1=BOC$, $2\sigma_2=COA$, $2\sigma_3=AOB$ $(\sigma_1+\sigma_2+\sigma_3=\pi)$, find the sum of the angles α, β, γ of the optical triangle ABCA, the order of the letters giving the sense of propa-

Fig. 4



gation. Denote by η_1 the angle between the optical ray CA and the radius vector at A, and let η_2, η_3 have analogous meanings for B and C, as in Fig. 4. Then, since the angle OCA is also equal η_1, and similarly for the remaining angles, we shall have

$$\alpha=\eta_1+\eta_2,\ \beta=\eta_2+\eta_3,\ \gamma=\eta_3+\eta_1, \text{ and}$$
$$\alpha+\beta+\gamma=2(\eta_1+\eta_2+\eta_3)\quad\ldots\ldots\ldots(5)$$

Now, by (2) and remembering that $A=r_o(1+r_o\omega)$, $\omega=\kappa\tilde{\omega}/c$,

$$r(\sin\eta+r\omega)=r_o(1+r_o\omega).$$

Apply this to the ray CA at the corner A. Then

$$\sin\eta_1=\frac{r_o}{a}(1+\omega r_o)-a\omega\quad\ldots\ldots\ldots\ldots\ldots\ldots(6)$$

and it remains only to find r_o. Now, the radius vector r_o bisects the angle $COA=\sigma_2$; thus, applying (3a), with $\theta_A-\theta_o=\sigma_2$, we have

$$\frac{r_o}{a}=\cos\sigma_2\left(1-a\omega\frac{\sin^2\sigma_2}{\cos\sigma_2}\right).$$

Substitute this in (6) and reject second order terms; then the result will be

$$\sin\eta_1=\cos\sigma_2-2\,a\omega\sin^2\sigma_2.$$

Since $\eta_1+\sigma_2$ differs but little from a right angle, put here $\eta_1=\frac{\pi}{2}-(\sigma_2+\delta_1)$; then the last equation will give for the small angle

$$\delta_1=2\,a\omega\,\sin\sigma_2.$$

Similarly the defects of $\eta_2+\sigma_3$ and $\eta_3+\sigma_1$ will be $\delta_2=2\,a\omega\,\sin\,\sigma_3$, $\delta_3=2a\omega\,\sin\sigma_1$. Thus by (5), and since $\sigma_1+\sigma_2+\sigma_3=\pi$,

$$\alpha+\beta+\gamma=\pi-2(\delta_1+\delta_2+\delta_3),$$

i.e., the *defect* of the optical triangle ABCA will be

$$4a\frac{\kappa\tilde{\omega}}{c}\,(\sin\sigma_1+\sin\sigma_2+\sin\sigma_3)\ldots\ldots(8)$$

For the optical triangle (or triangular circuit) ACBA we shall have an equal *excess* of the angle sum.

Thus, for instance, for an equilateral triangle, $\sigma_1=\sigma_2=\sigma_3=60°$, the defect, or the excess, will be $4a\frac{\kappa\tilde{\omega}}{c}.\,3\,\frac{\sqrt{3}}{2}=6\sqrt{3}a\frac{\kappa\tilde{\omega}}{c}$. For the Earth (and a triangle parallel to the equatorial plane) even if $\kappa=1$ (no drag) this would amount to 0.″00052 per kilometer of a, and the difference between the angle sum of ACBA and ABCA would be the double of this. Thus, even for $a=10$ or 20 km. the difference would certainly be too small to be measured directly.

The experimental possibilities with regard to the optical effects of the rotation of the Earth lie in another direction, to wit in the phase retardation in an optical circuit (i.e., closed light path) as in the well-known laboratory experiment of Sagnac[4] with a small spinning interferometer as our system S. The corresponding formula used by Sagnac and before him by Michelson (Phil. Mag. vol. 2, 1904, p. 716–719) who actually proposed but never carried out a terrestrial experiment[5] of the kind here aimed at, can be

---

[4] G. Sagnac, "L'ether lumineux démontré par l'effet du vent relatif d'ether dans un intérferomètre en rotation uniforme." C. R. Paris, 157 (1913), p. 708. Ibidem, p. 1410, "Sur la preuve de la réalité de l'ether lumineux par l'experience de l'interférographe tournant." The titles seemed interesting enough to be quoted in full. But Sagnac's experiments (even apart from the question of the reliability of his measurements) by no means decide for the aether as against relativity.

[5] Such a terrestrial experiment was already hinted at by Oliver Lodge, in 1897, Phil. Trans. Roy. Soc. A, vol. 189, p. 151, where also the experiment carried out by Sagnac twenty-five years later is suggested, only with "telescope and observer" instead of a photo camera mounted on a rotating "turn-table."

most simply deduced in the following way. Consider any optical circuit s, traversed by light in the sense of rotation or "positive" circuit, say.[6] By formula (1), the time taken to traverse an element ds of the circuit is, with u_s written for $\mathbf{up}$,

$$ds.(c+u_s)^{-1}=\frac{ds}{c}\left(1-\frac{u_s}{c}\right),$$

giving a retardation $-\frac{1}{c^2}u_s\,ds$ per line-element, and thus for the whole circuit the phase retardation

$$\Delta\tau=-\frac{1}{c^2}\int \mathbf{u}\,d\mathbf{s},$$

as already noticed by Lorentz (Wiss. Abhandlungen, vol. I). Now, by Stokes' theorem, this can be written, if σ be any surface laid through s, and $\mathbf{n}$ its normal,

$$\Delta\tau=-\frac{1}{c^2}\int \mathbf{n}\ \mathrm{curl}\ \mathbf{u}.\ d\sigma.$$

This holds for any circuit, plane or not, and for any distribution of velocity. Since $-\mathrm{curl}\ \mathbf{u}$ is the double angular velocity, $2\kappa\tilde{\omega}.\mathbf{k}$, we have, writing $\tilde{\omega}_n=\tilde{\omega}\mathbf{kn}$ for the normal component of the spin,

$$\Delta\tau=\frac{2}{c^2}\int \kappa\tilde{\omega}_n d\sigma \qquad (9)$$

This, the required formula, is valid for any, not necessarily constant value of $\kappa\tilde{\omega}$ throughout the surface of integration. In our case, $\tilde{\omega}$, and therefore $\tilde{\omega}_n$ for a plane circuit, are manifestly constant, but the drag of the aether, if any, may vary from point to point, thus giving rise to a variable coefficient κ.

If the optical circuit is plane and small compared with the dimensions of the Earth, we have simply $\Delta\tau=2\kappa\tilde{\omega}_n\sigma/c^2$, or if T be the period of oscillation and $\lambda=cT$,

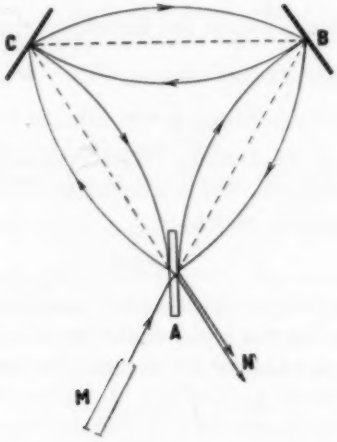
$$\frac{\Delta\tau}{T}=\frac{2\kappa\tilde{\omega}_n}{c\lambda}\sigma \qquad (10)$$

where σ is the total area embraced by the circuit. This gives the retardation, in parts of the period, for a positive circuit. As was already mentioned, the same path cannot, rigorously, serve for light propagation in the opposite sense [or, which is the same

[6] As we already know, the same light path cannot be described in the opposite sense.

thing, with inverted normal $\mathbf{n}$ in (10)]. Thus, for instance, in the case of a triangle (Fig. 5) we have to take for the positive circuit the area σ_1 of the inner, and for the negative circuit the area σ_2 of the outer, convex triangle, so that rigorously we would

Fig. 5



have for the phase difference of the two beams (say, separated at A by a semi-transparent plate and reflected at B, C) in parts of the period, or for the corresponding shift in fringe widths,

$$\epsilon=\frac{2\kappa\tilde{\omega}}{c\lambda}(\sigma_1+\sigma_2) \qquad (11)$$

But the difference of σ_1 and σ_2 is itself small of the first order. Moreover, up to higher terms, the paths AB and BA, etc., are symmetrical with respect to the corresponding (dotted) straights, so that even up to terms of an order higher than the second we can replace $\sigma_1+\sigma_2$ by trice the area (σ) of the rectilinear (dotted) triangle, and similarly in the case of any polygons. Thus:[7]

$$\epsilon=\frac{4\kappa\tilde{\omega}_0\sigma}{c\lambda} \qquad (12)$$

[7] Prof. Michelson's paper of 1904, (l.c.) has by a manifest slip the factor 2 instead of our 4.

For horizontally placed circuits we have $\tilde{\omega}_n = \tilde{\omega} \sin \varphi$, if φ be the geographic latitude at which the experiment is performed. For a latitude $\varphi = 45°$ and the wave-length $\lambda = 5000$ Å. U., formula (12) gives

$$\epsilon = 1.38\kappa \frac{\sigma}{\text{km}^2} \quad \text{.....(12a)}$$

i.e., about 1.4κ fringe widths per each square kilometer embraced by the circuit. If the drag is complete ($\kappa = 0$) there should be no shift; if there is no drag, we should have the total amount of 1.4 per km², and intermediate values if the Earth in its spinning motion drags the aether, even at its surface, only partially.

In Sagnac's experiment the spinning motion of the (disc-shaped) table bearing the interferometer, the light source, as well as the photographic camera, was reversed, and thus the double of (12) was observed[8] as the shift of the system of interference fringes. No such reversal, of course, is possible in the case of the Earth as the rotating system. But as Prof. Michelson has already pointed out in 1904 (loc. cit.) there is an easy way out of this difficulty, to wit by silvering heavily one, e.g., the upper, half of the dividing glass plate (such as A in the case of Fig. 5 corresponding to three stations; B, C, being mirrors) and leaving the lower part clear or but lightly silvered. A beam of parallel rays from the collimator M impinging upon the plate A is here divided into ACBA and ABCA by reflection and transmission respectively. Now, according to Michelson's suggestion, cover the lower half and observe first by reflection from the upper half only (ACBA) when simply the image of the slit is seen. Place the cross-hair of the eyepiece in the center of this image. Next, covering the upper half and leaving the lower half of the plate clear, observe the interference fringes. Then, if there is an effect ϵ, the midpoint of the central fringe will be displaced from the crosshair by ϵ fringe widths. The effect sought for will be easily discernible from undesired accessorial shifts by being proportional to the area σ. Instead of comparing the position of the interference

[8] In Sagnac's case $\kappa = 1$, since such small masses as was his table certainly do not drag the aether, as follows from the widely known older experiments of Sir Oliver Lodge.

fringes with that of the slit image Prof. Michelson contemplates also the comparison of the two fringe systems given by two considerably differing values of the area σ.

It is estimated that the terrestrial experiment could be carried out with the required precision on comparatively small areas, such as 1/10 of a square kilometer. The beams can, of course, be sent around twice or more times making ϵ as many times larger; as many times, however, would the light path be increased which may not be convenient. A more radical way is to leave the circuits simple but to extend the linear dimensions of the circuits; for then the value of ϵ will be increased in the squared ratio. Again the more stations (arranged in a regular polygon) the greater σ for the same light paths. Yet, to avoid too many reflections and other inconvenients, the triangular arrangement as suggested in Fig. 5 or a quadrangular one may turn out to be preferable. But details of a technical kind need not detain us in the present paper.

2. Let us now try to find out what aspect the same problem assumes from the standpoint of the theory of relativity, the special and the generalized one. It goes without saying that with neither of these theories can there be any question of an aether and its being dragged by the Earth in its daily rotation around its axis, or in its annual motion around the sun.

First of all, then, the special relativity theory is wholly incompetent to deal with the problem of light propagation in a rotating system rigorously, simply because it has nothing to do with any reference systems other than the galilean or inertial ones, i.e. the fixed-stars system S* and those moving relatively to it with uniform translational velocity. The only thing the special theory can do is to treat our problem approximately up to the second order, i.e., rejecting the square (and higher powers) of the ratio β of the velocity of motion[9] to the light velocity. Now up to β^2 the relativistic addition theorem of velocities does not differ at all from that of ordinary, Newtonian kinematics. This amounts simply to a neglect of the Lorentz-Fitgerald contraction. Under

[9] Of any point of S relatively to S* say.

these circumstances the propagation of light in S becomes simply a superposition of that in S* and of the reversed spinning motion of S relatively to S* in much the same way as on the aether theory. The only difference is that our previous $\kappa\tilde{\omega}$ has now to be replaced by *the full angular velocity* $\tilde{\omega}$ of S relatively to S* (fixed stars). Thus formula (1) for the light velocity v will be replaced by

$$\frac{v}{c}=1+\frac{1}{c}u_s=1+\frac{\tilde{\omega}r}{c}\cos\gamma,$$

and the shift formula (12) by

$$\epsilon=\frac{4\tilde{\omega}_n\sigma}{c\lambda} \qquad (12r)$$

In fact v. Laue, the chief exponent of Einstein's older theory in Germany, gives in his well-known book, in a section on Sagnac's experiment, a formula identical with (12r) at which he arrives by a rather roundabout way, instead of using the simple relation (9), valid for a circuit of any shape.[10]

Similarly all other formulae given above will continue to hold on the special relativity theory with unity written for κ.

From the standpoint of the general relativity theory all questions concerning light propagation (in vacuo) relatively to any system whatever and therefore also to our terrestrial system S will be answered if the four-dimensional line-element $ds^2=\Sigma g_{i\kappa}\,dx_i dx_\kappa$ belonging to that system be known, to wit, by putting

$$ds=0 \qquad (13)$$

Thus the problem is reduced to building up ds in terms of terrestrial or S-co-ordinates, say $x_1, x_2, x_3, x_4=r, \theta, z, ct$, respectively, keeping in mind that in the fixed-star system S* (disregarding in our present connection the extremely minute terms due to the Earth's gravitation) the line-element is

$$ds^2=c^2dt'^2-dr'^2-r'^2d\theta'^2-dz'^2 \qquad (14^*)$$

To be faithful to its own leading principles, the relativity theory ought to deduce the terrestrial ds or its coefficients $g_{i\kappa}$ as a gravi-

[10] M. v. Laue, Relativitätstheorie, vol. I, 3rd ed. Braunschweig, 1919, p. 125–127. Laue seems, by the way, to be under the misapprehension that the light rays relative to the rotating table are straight lines, which they are not. As we saw before, their departure from straight lines is a first order effect and does, therefore, by no means disappear though "the Lorentz contraction" be neglected. The influence of the curvature of the rays on (12) or (12r) remains, of course, negligible.

tational effect of the stars and all the remaining matter of the universe rotating around the Earth. This, however, it proved unable to do. As a matter of fact Einstein himself never entered into the details of this important problem of rotation. Thirring[11] tried to solve it by considering a huge massive spherical shell in uniform rotation and evaluating by means of Einstein's approximate integrals of his field-equations the $g_{\mu\kappa}$ thus produced at comparatively small distances from the centre of that gigantic sphere. But the result, though mathematically interesting, was a complete failure,[12] although the treatment of the same problem on Einstein's newest cosmological views (elliptic space and so on) seems more promising. At any rate, the relativity theory is unable to construct the required line-element on its own great principles, and is content to transcribe it from the galilean line-element (14*) by putting simply $r', z', t' = r, z, t$, and

$$\theta' = \theta + at,$$

where a is an appropriate constant. In fact, de Sitter, one of the chief exponents of Einstein's theory, and even Weyl in his interesting book[13] write down without much discussion this simple transformation in order to pass from one to the other system. This gives for the latter,

$$ds^2 = \left(1 - \frac{a^2r^2}{c^2}\right)c^2dt^2 - 2r^2\frac{a}{c}d\theta cdt - dr^2 - r^2d\theta^2 \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

The constant a is to be chosen so as to describe correctly the terrestrial laws of physical phenomena, approximately, at least, in our present connection up to the second order only.

Now, if but such an approximation is required, the fundamental principles of general relativity offer us a good test of whether a chosen value of the constant a is or is not the appro-

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[11] H. Thirring, Ueber die Wirkung rotierender ferner Massen in der Einsteinschen Gravitationstheorie. Phys. Zeitschrift, Vol. 19, 1918, p. 33–39.

[12] Not only that Thirring's "centrifugal force" had also a component along the axis of rotation, but the coefficients of the centrifugal and the Coriolis force, apart from being very unsatisfactory in their structure, bore a wrong numerical ratio to one another.

[13] H. Weyl, Zeit-Raum-Materie, 3rd ed., Berlin, J. Springer, 1920.

priate one. In fact, according to one of these principles $ds=0$ represents light propagation, and according to the other the geodesics of the world, determined by the same line-element, i.e.,

$$\delta\int ds=0 \qquad (15)$$

give the equations of motion of a free particle. And since the terrestrial laws of motion are certainly known to that degree of approximation, we can derive from (15) the correct value to be attributed to the constant a.

Now, using (14) in (15) the terrestrial equations of motion of a free particle (always apart from gravitation proper) follow at once. Of these equations it will be enough to write down only that which most interests us here, i.e., the equation corresponding to the variation δr. This is

$$\frac{d^2r}{ds^2}=a^2r\left(\frac{dt}{ds}\right)^2+2ar\frac{d\theta}{ds}\frac{dt}{ds}+r\left(\frac{dt}{ds}\right)^2,$$

and since, up to the second order, $ds^2=c^2\,dt^2$, we can write

$$\frac{d^2r}{dt^2}=r\left(a+\frac{d\theta}{dt}\right)^2.$$

The right-hand member will represent the correct value of the centrifugal acceleration (or "centrifugal force" per unit mass), in size and direction, provided that a stands for the *full* angular velocity of the Earth relatively to the fixed stars, i.e.

$$a=\bar{\omega}.$$

This then is the value to be substituted into (14) to suit our dynamical experience, and at the same time, as already explained, into the light equation $ds=0$. In short, on the relativity theory the same ds and therefore the same $\bar{\omega}$ is required for light as for mechanics, while on the aether theory we may have any fraction $\kappa\bar{\omega}$ of $\bar{\omega}$ for light, though the full angular velocity $\bar{\omega}$ is required for mechanics.

Thus, on the relativity theory the terrestrial velocity of light for any direction of the ray will be determined by

$$\left(1-\frac{\bar{\omega}^2r^2}{c^2}\right)c^2dt^2-2\frac{\bar{\omega}}{c}r^2d\theta cdt-dr^2-r^2d\theta^2=0 \qquad (16)$$

whence also the form of the optical rays and the circuitous shift

formula can be deduced.[14] Or, equivalently and much more simply, we may deduce all these terrestrial optical laws from those of S* (uniform propagation with velocity c) by means of the transformation

$$\theta' = \theta + at = \theta + \tilde{\omega}t.$$

Consequently, all our previous formulae for the light velocity, for the shape of rays and the phase retardation in a terrestrial optical circuit, will reappear, with the only difference that the unknown fraction κ will be replaced by the special and definite value 1, i.e., our $\kappa\tilde{\omega}$ will be replaced by $\tilde{\omega}$. Thus also the shift formula, which from the experimental point of view is the only important thing, will again be

$$\epsilon = \frac{4\tilde{\omega}_n\sigma}{c\lambda},$$

as on the special relativity theory.

The planned terrestrial experiments with the optical circuit might thus enable us to decide between relativity and aether theory. That is to say, if the result of such experiments will be a full effect (a shift of 1.38 fringe widths under the stated conditions, per square kilometer), there will be no discrimination between the two theories. But should there be either no shift at all or only a fraction κ of the full effect, sensibly different from unity, the relativity theory, special or general, will be irremediably disproved, while on the aether theory we would have only to assume a complete or partial rotational dragging of the aether.

In fine, the optical circuit experiment may easily become crucial and fatal for Einstein's theory especially if it gives a nil-effect. Should it, on the other hand, give a full effect, it will certainly cease to be decisive either way but will even then (as every new positive experiment) be certainly a valuable contribution

[14] The only effect of terrestrial gravitation on the optical-circuit experiment would according to relativistic gravitation theory, be represented by the term $\frac{8}{5}\cos^2\varphi\frac{M}{c^2R}$ to be subtracted from the factor 2 of the second term in (16), φ being the geographic latitude, M the mass and R the radius of the Earth. But since $M/c^2 = 0.45$ cm., the correction term due to gravitation amounts, even at the equator, only to 1.2×10^{-9} which is entirely negligible in presence of 2.

to our experimental knowledge of the Earth as a physical reference system.

Speculations on a possible rotational drag of the aether by the Earth were made, in connection with the diurnal aberration, by Christian Doppler (1845) and after him by v. Oppolzer.[15] But owing to an almost complete absence of direct observational data on daily aberration, a state of things prevailing even at the present time, Doppler's formula, generalized by Oppolzer, could neither be proved nor disproved. More recently the question of a rotational drag was taken up by Lorentz[16] but it had again to be left unsettled in absence of daily aberration data. If there is anything on the experimental side to make the rotational drag unlikely it is the absence of shift of a star near occultation by Jupiter, as mentioned by Lorentz (loc. cit. p. 413). On the theoretical side the question of the influence of a *spinning* planet on the adjacent aether could not be answered definitely without some special and more or less artificial hypotheses about the properties of that medium in addition to those given to it already by Stokes-Planck. In this respect we can say only that even if there is an almost complete translational (annual) drag of the aether due to its condensation around the planet, there may yet be no appreciable rotational (daily) drag. Again, from Lodge's experiments (loc. cit) one can judge only that there is no such drag by comparatively small spinning masses, such as can be used in a laboratory, but not by such massive bodies as the Earth or other planets. In fine, the only sound way of settling the question would be to carry out the terrestrial experiment with optical circuits embracing as large an area as is technically possible.

ROCHESTER, N. Y.
March, 24, 1921.

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[15] E. v. Oppolzer, Erdbewegung and Aether, Sitzber. Akad. Wien, vol. CXI, IIa, Febr. 1902, pp. 244–254.

[16] H. A. Lorentz, Abhandlungen, Vol. I.

STANDARD WAVE-LENGTHS*

BY
W. F. MEGGERS

I. INTRODUCTION

A train of waves emitted by a source of homogeneous light represents a scale of invariable quality, whose use in making physical length measurements of extraordinary precision is now more or less familiar to everyone. For practical use, it is necessary to choose very narrow spectral lines and to define the lengths represented by their waves and the conditions producing them. Several attempts have been made to define such a system of standard wave-lengths, each successive attempt aiming at higher precision by making use of more satisfactory light sources and more refined methods of measurement. At the present time, it appears likely that the question will be reopened and changes in the pre-war international code of standard wave-lengths have already been suggested. The importance of the fundamental standards to spectroscopy, metrology and precision optics generally, makes it imperative that this subject be brought to the attention of the Optical Society of America. A complete understanding of the present tendency in this field is not possible, however, without some knowledge of the history of standard wave-lengths; this report therefore presents a summary of the past activities in this field as an introduction to the present status and tendency of standard wave-lengths.

II. HISTORICAL REVIEW

1. ROWLAND SYSTEM OF STANDARDS

The first system of standard wave-lengths of importance was the Preliminary Table of Solar Spectrum Wave-Lengths given by Rowland in 1888. For twenty years, practically all spectroscopic measurements were based on these values and even at the present time they are widely used by astrophysicists. The system was based upon an absolute value of the wave-length of D_1 of

* 1920 Report of Committee on standard wave lengths.

sodium (5896.156) adopted by Rowland[1] as the weighted mean of various determinations, and consisted of a small number of lines compared with this one by the method of coincidences in diffraction grating spectra, while the remainder were measured by interpolation. Rowland thought that the absolute values were correct to 1 part in 100,000 and that the errors in relative value were not greater than a millionth.

In 1893, the absolute measurements of Michelson & Benoit on cadmium lines in terms of the standard meter at Paris showed that Rowland's wave-lengths were too large by 1/30,000 but spectroscopists paid little attention to this since there was still no reason to doubt the correctness in relative value of Rowland's numbers. In 1901, Fabry and Perot[2] measured about 30 lines in the solar spectrum between 4643 A and 6471 A by means of their interferometer. This work revealed the fact that systematic errors existed in Rowland's table so that the relative values of wave lengths in the interval which was studied contained errors of nearly 1 part in 100,000. These errors were attributed to the erratic behavior of diffraction gratings and Kayser[3] made some experiments in 1904 with Rowland gratings which led him to the conclusion that the coincidence between orders of spectra with gratings is not to be depended upon. Miss Howell[4] has since shown that the fundamental law holds for good gratings so that the errors in Rowland's values as well as the experience of Kayser may require some other explanation. Jewell[5] has stated that the relative errors in Rowland's Table are due to disturbed apparatus and failure to make any corrections for Doppler effect or for air-pressures and temperatures, while Goos[6] has shown that most of the discrepancies observed by Kayser are explained by slight defects in the adjustment of the grating.

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[1] Phil. Mag. (5), 23, p. 257; 1887, and 27, p. 479; 1889.
[2] Astroph. J. 15, p. 73 and 261; 1902.
[3] Astroph. J., 19, p. 157; 1904.
[4] Astroph. J., 39, p. 230; 1914.
[5] Trans. I. U. S. R., 1, p. 42; 1906.
[6] Zeit. f. Wiss. Phot., 9, p. 173; 1911.

2. INTERNATIONAL SYSTEM OF STANDARDS

The question of standard wave-lengths was discussed at the first meeting of the International Union for Co-operation in Solar Research[7] and the necessity for establishing a new system of standard wave-lengths taken from artificial sources was agreed upon.

At the second meeting[8] of the Union in 1905, the following decisions were reached. (1) The wave-length of a properly chosen radiation shall be taken as a primary standard of wave-lengths. The number which represents this wave-length shall be fixed one time for all; it shall define the unit of wave-length which must differ as little as possible from 10^{-10} meter and shall be called angstrom. (2) There is reason to choose secondary standards separated by not more than 50 angstroms. These secondary standards shall be referred to the primary standard by a method of interference. The luminous source shall be an electric arc of 6 to 10 amperes.

a. *The Primary Standard*

In 1907, a report of the measurement by Benoit, Fabry and Perot[9] of the wave-length of the red radiation from cadmium relative to the standard meter was presented at the third meeting of the International Union and the following resolution was adopted: "The wave-length of the red ray of light from cadmium produced by a tube with electrodes is 6438.4696 Angstroms in dry air at 15° on the hydrogen thermometer, at a pressure of 760 mm of mercury, the value of g being 980.67 (45°). This number will be the definition of the unit of wave-length." The wave-length of the red radiation of cadmium thus defined in terms of the standard meter was claimed to be correct within 1 ten millionth which is also the limit of accuracy attainable in the intercomparison of meter bars. The value differs by less than 1 part in 16 million from the corrected value obtained by Michelson 14 years earlier.

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[7] Astroph. J., 20, p. 313; 1904.
[8] Trans., I.U.S.R., I, p. 234; 1906.
[9] Trans., I.U.S.R., II, p. 109; 1907.

b. *The Secondary Standards*

In 1907, the first determinations of secondary standards were reported (reference 9, p. 138) by Fabry and Buisson who measured the wave lengths of 115 lines in the iron arc 2373–6494 by means of their interferometer method. Similar measurements were made by Eversheim[10] and by Pfund,[11] and the mean values of three independent determinations were adopted as secondary

TABLE I

International Secondary Standards

3370.789	4375.934	5405.780
3399.337	4427.314	5434.527
3445.154	4466.556	5455.614
3485.345	4494.572	5497.522
3513.821	4531.155	5506.784
3556.881	4547.853	5569.633
3606.682	4592.658	5586.772
3640.392	4602.947	5615.661
3676.313	4647.439	5658.836
3677.629	4691.417	5709.396
3724.380	4707.288	5763.013
3753.615	4736.786	5857.759 Ni
3805.346	4789.657	5892.882 Ni
3843.261	4859.758	6027.059
3850.820	4878.225	6065.492
3865.527	4903.325	6137.701
3906.482	4919.007	6191.568
3907.937	4966.104	6230.734
3935.818	5001.881	6265.145
3977.746	5012.073	6318.028
4021.872	5049.827	6335.341
4076.642	5083.344	6393.612
4118.552	5110.415	6430.859
4134.685	5167.492	6494.993
4147.676	5192.363	6546.250
4191.443	5232.957	6592.928
4233.615	5266.569	6678.004
4282.408	5302.315	6750.163
4315.089	5324.196	
4352.741	5371.495	

[10] Ann. d. Phys., (4), 36, p. 1071; 1911.
[11] Astroph. J., 28, p. 197; 1908.

standards by the International Union in 1910, comprising 54 lines between 4282 A and 6484 A. Additional values of international secondary standards were adopted from time to time,[12] and the complete list is given in Table 1.

The agreement between interference observations of these wave lengths is remarkably good and there is reason to believe that most of the accepted secondary standards have a precision of 0.001 A, i.e., they are correct and reproducible to 1 part in several millions. This is the extreme limit which it is possible to obtain since the most homogeneous lines of iron have a width of about 0.060 A and a precision of 0.001 A therefore corresponds to 1/60 of the width of the line. Some of the differences between values of different observers can probably be accounted for by different arc conditions employed in different laboratories and special investigations on the effect of arc conditions showed that the wave-lengths of certain iron lines which are sensitive to pressure are also affected by current strength, and by the length and portion of arc used.[13] Recognition of these influences led, in 1913, to the adoption of more precise specifications of the iron arc in air[14] as a source of international standards as follows: (1) Length of arc 6 mm; (2) Current of 6 amperes for wave-lengths greater than 4000 A, for wave-lengths less than 4000A, 4 amperes or less; (3) Use direct current with positive pole above the negative and a potential of 220 volts; iron rods of 7 mm diameter; (4) As a source of light use an axial part of about 2 mm in the middle of the arc; (5) Use only iron lines of groups a, b, c, d (Mt. Wilson Classification).

In the past ten years, very little progress has been made in extending this system of secondary standards. The most extensive measurements were made by Burns, who compared directly with the cadmium primary standard, the wave-lengths of 125 iron lines[15] from 5434 A to 8824 A and 100 lines[16] from 2851 A to 3701 A.

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[12] Astroph. J., 32, p. 215, 1910. Ibid. 33, p. 85, 1911; Ibid. 39, p. 93; 1914.

[13] St. John and Ware, Astroph, J., 36, p. 14; 1912 and 39, p. 5; 1914. Goos, Astroph. J., 38, p. 141; 1913.

[14] Trans., I.U.S.R., 4, p. 58; 1914.

[15] Jour. de Phys. (5), 3, p. 457; 1913.

[16] B. S. Bulletin 12, p. 179; 1915.

A few red lines were measured by Eversheim[17] but his values and those of Burns diverge rather widely for the longer waves.

c. *Tertiary Standards, Pole Effect*

In addition to the above determinations of secondary standards, many additional measurements of wave-lengths in the iron spectrum have been made, some with interferometers and others with diffraction gratings, partially to set up a system of tertiary standards and also to investigate further the effect of operating conditions of the arc upon its wave-lengths. Among such investigations the most important in some respects appears to be that of the so-called "pole effect" on arc lines. This has been studied in some detail at the Mt. Wilson Observatory[18] where it is found that certain iron lines which are sensitive to pressure have slightly different wave-lengths in the center of the arc than near the negative pole (one or two millionths), and also that this effect is somewhat reduced when, instead of using two iron electrodes, a carbon electrode is used with an iron one.[19]

III. RECENT DEVELOPMENTS

1. NEW INTERNATIONAL UNION

The world war destroyed the old international co-operation but a new International Astronomical Union, representing several of the allied countries, was organized in Brussels in July 1919. The question of Standard Wave-Lengths was revived by a committee appointed by the American Section of the then proposed International Astronomical Union, meeting at Washington March 8, 1919; from whose report[20] of memoranda for the American delegates to the first meeting of the new Union the following paragraphs are quoted:

The Primary Standard. The remarkable sharpness of the red cadmium line, the extraordinary skill with which the meter was evaluated in terms of this wave-length, and the final definition of the angstrom, as an arbitrary unit, by the International Solar Union at Paris in 1907 would seem to place the primary standard almost beyond the range of question.

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[17] Ann. der Phys., 45, p. 454; 1914.

[18] St. John and Babcock, Astroph. J., 42, p. 231; 1915.

[19] St. John and Babcock, Astroph, J., 46, p. 138; 1917.

[20] Proc. Nat. Acad. Sci., 6. p. 367; 1920

However, the recent perfection of the new method for making end-standards of length, the comparative ease with which the optical length of these end-standards can be determined, and the possible superiority of some other line (such as the bright yellow neon line) over one from solid cadmium as a source of light combine to urge a reconsideration even of the primary standard.

Sources. In order to obtain lines of constant wave-length and constant intensity distribution, it is recommended that the modification of the Pfund arc employed at Mt. Wilson be adopted, namely, the Pfund arc operated under 120–240 volts, with 5 amperes or less, at a length of 12 millimeters, and used over a central zone, at right angles to the axis of the arc, not to exceed 1¼ mm in width.

These sections are the only ones which suggest fundamental changes in the erstwhile International System of Standard Wave Lengths, and their validity can be determined only by careful consideration and decision that the expected improvements will justify both the loss of enormous labor which produced the present system and the additional task of building up a new one.

2. THE PRIMARY STANDARD

Exact reproducibility and high homogeneity are the most important characteristics of a primary standard wave length for purposes of precision measurement. There is little or no reason to doubt the reproducibility of the wave-length of the cadmium red line. As nearly as can be determined the same wave-length has been observed from tubes of various designs, with or without electrodes and operated with different currents,[21] pressures and purity of metal.

The total width of a spectral line, assuming that it is accounted for by the Doppler-Fizeau effect, has been shown to be

$$\Delta = 0.82 \times 10^{-6} \times \lambda \sqrt{\frac{T}{M}}$$

where λ represents the wave-length, T the absolute temperature and M the molecular weight of the radiating gas or vapor. The reciprocal of the relative value $\frac{\Delta}{\lambda}$ defines the limiting order of interference N which the width of the line permits, and may be called the sharpness or fineness of the line.

$$N = 1.22 \times 10^{6} \sqrt{\frac{M}{T}}$$

[21] Trans., I.U.S.R., 2, p. 24; 1907.

This has been verified experimentally by Buisson and Fabry.[22] Michelson[23] actually determined the widths of many spectral lines when searching for a homogeneous radiation with which to compare the meter, and the red line in the spectrum of cadmium was the narrowest line then known. Only in recent years have sharper spectral lines been found. This is in consequence of the discovery and study of the so-called rare or inert elements which exist as gases not only at ordinary and high temperatures but even at very low temperatures. These gases have a range in molecular weights from 4 to 222 and can be made to emit rather complicated spectra of great intensity.

If a line from one of the rare gas spectra replaces the cadmium line as a fundamental standard, it should be chosen from one of the heavy gases. At the temperatures usually employed in their production, the neon lines are not as sharp (N = 320,000) as the cadmium line (N = 500,000). The green line 5570 A in the spectrum of krypton, however, has a width of only 0.006 A and its limiting order of interference has been observed as 600,000 at ordinary temperatures and almost a million near the temperature of liquid air. Theoretically, the lines in the spectrum of niton are nearly twice as sharp as those of krypton. Perhaps the chief obstacle to the present use of such lines is that the heavy gases are extremely rare and Geissler tubes of them are practically unobtainable. The recent disclosure of relatively enormous quantities of helium gas in natural gas wells suggests the possibility of also finding larger quantities of the heavier gases. At the present time it is perhaps doubtful if the possible increase in precision is so necessary as to warrant the adoption of a light source which is almost inaccessible and which would require repeating the difficult experiment of comparing a new wave-length with the meter.

3. SECONDARY STANDARDS

Since the spectral lines from Geissler tubes of the inert gases are exceptionally sharp, reproducible and convenient to use, their wave-lengths can be determined very accurately and may

[22] J. de Phys., April 19, 1912.
[23] Phil. Mag., (5), 31, p. 338; 1891.

serve excellently as standards. For several years the wave-lengths of helium and neon lines have been in continuous use at the Bureau of Standards for the accurate length measurements of precision end standards and munition gages and for a study of the doubtful permanency of such material standards.[24]

The wave-lengths of many lines in the spectra of helium, neon, argon, krypton and xenon have already been compared with the primary standard (6438.4696A) by interferometer methods and it would appear advisable to adopt such lines as additional international standards in all cases where three observers are in good agreement. A summary of these data will now be given.

TABLE 2

Standard Wave Lengths from Helium

Bureau of Standards[25]	Rayleigh[26] a	Rayleigh[26] b	Eversheim[27]	Ignatieff[28]
2945.104	....	....	....	....
3187.743	....	....	....	....
3613.641	....	....	....	....
3705.003	....	....	....	....
3819.606	....	....	....	....
3888.646	....	....	....	....
3964.727	....	....	....	....
4026.189	....	....	....	....
4120.812	....	....	....	....
4143.759	....	....	....	....
4387.928	....	....	....	....
4471.477	(.478)	.480	.493	....
4713.143	(.171)	.142	.154	....
4921.929	.925	.928	.934	....
5015.675	.680	.678	.683	....
5047.736	....	....	....	....
5875.618	.616	.623	.639	....
6678.149	.144	.147	.151	....
7065.188	.189	.197	.207	....
7281.349	....	....	....	....
....	....	....	....	10829.11
....	....	....	....	10830.32
....	....	....	....	20581.31

[24] Peters and Boyd, American Machinist, Sept. 30 and Oct. 7, 1920.
[25] Merrill, B. S. Bulletin 14, p. 159; 1917.
[26] Phil. Mag. (6) 11, p. 685; 1906. Ibid (6) 15, p. 548; 1908.
[27] Zt. f. wiss. Phot., 8, p. 148; 1909.
[28] Ann. d. Phys (4) 43, p. 1117; 1914, relative to 5015.6815.

TABLE 3

Standard Wave-Lengths from Neon

Bureau of Standards[29]	Meissner[30]	Bureau of Standards[31]
3369.904	.....	.....
3417.906	.....	.....
3447.705	.....	.....
3454.197	.....	.....
3460.526	.....	.....
3464.340	.....	.....
3466.581	.....	.....
3472.578	.....	.....
3498.067	.....	.....
3501.218	.....	.....
3515.192	.....	.....
3520.474	.....	.....
3593.526	.....	.....
3593.634	.....	.....
3600.170	.....	.....
3633.664	.....	.....
5330.779	.....	.....
5341.096	.....	.....
5400.562	.564	.....
5764.419	.....	.....
5820.155	.....	.....
5852.488	.4875	.4862
5881.895	.896	.8958
5944.834	.834	.8344
5975.534	.534	.....
6029.997	.999	.....
6074.338	.337	.3383
6096.163	.163	.1608
6143.062	.061	.0600
6163.594	.594	.....

[29] Burns, Meggers and Merrill, B. S. Bulletin 14, p. 765; 1918.
[30] Ann. d. Phys. (4) 58, p. 333; 1919, and 60, p. 414; 1919.
[31] Priest, B. S. Bulletin 6, p. 573; 1911. Ibid. 8, p. 539; 1912.

TABLE 3 (*Continued*)
Standard Wave-Lengths from Neon

Bureau of Standards	Meissner[30]	Bureau of Standards[31]
6217.280	.279	.....
6266.495	.495	.4948
6304.789	.788	.7929
6334.428	.428	.....
6382.991	.991	.9882
6402.245	.2460	.2392
6506.528	.527	.....
6532.883	.881	.....
6598.953	.953	.....
6678.276	.275	.....
6717.043	.042	.....
6929.468	.465	.....
7024.049	.....	.....
7032.413	.410	.....
7051.292	7051.314	.....
7059.109	7059.119	.....
7173.938	7173.938	.....
7245.166	7245.165	.....
7438.899	7438.885	.....
7488.872	.....	.....
7535.784	7535.786	.....
7544.050	7544.061	.....
.....	7937.010	.....
7943.182	7943.193	.....
.....	8082.460	.....
.....	8118.554	.....
8136.408	8136.423	.....
.....	8259.392	.....
.....	8266.092	.....
8300.369	8300.338	.....
8377.606	8377.630	.....
.....	8418.447	.....
8495.358	8495.359	.....
.....	8591.266	.....
.....	8634.668	.....
.....	8654.380	.....

TABLE 4

Standard Wave-Lengths from Argon

Bureau of Standards[22]	Meissner[23]	Bureau of Standards[22]	Meissner[23]
3948.980	.....	7147.042	.....
4044.419	.....	7206.986	.....
4158.591	.....	7272.935	.....
4164.180	.....	7353.316	.....
4181.884	.....	7372.119	.....
4190.714	.....	7383.979	978
4191.027	.....	7503.867	868
4198.316	.....	7514.651	648
4200.676	.....	7635.106	107
4251.184	.....	7723.758	760
4259.362	.....	7724.210	210
4266.286	.....	7948.175	177
4272.169	.....	8006.156	158
4300.101	.....	8014.784	786
4333.561	.....	8103.693	691
4345.168	.....	8115.307	310
4510.733	.....	8264.522	525
4522.325	.....	8408.210	216
4596.096	.....	8424.646	650
4628.445	.....	8521.443	.....
4702.317	.....	.....	.....
6032.127	.....	.....	.....
6416.307	.....	.....	.....
6677.282	.....	.....	.....
6752.831	.....	.....	.....
6871.290	.....	.....	.....
6937.666	.....	.....	.....
6965.429	432	.....	.....
7030.250	.....	.....	.....
7067.217	218	.....	.....

[22] Meggers, To be published in B. S. Bulletin.

[23] Ann. d. Phys. (4) 51, p. 95; 1916.

TABLE 5

Standard Wave-Lengths from Krypton

Bureau of Standards[34]	Buisson and Fabry[35]
4273.9696	.....
4282.967	.....
4318.552	.....
4319.580	.....
4355.478	.....
4362.6422	.....
4376.122	.....
4399.969	.....
4453.9174	.....
4463.690	.....
4502.354	.....
4807.065	.....
5562.224	.....
5570.2872	.2908
5870.9137	.9172
6456.290	.....
7587.414	.....
7601.544	.....

TABLE 6

Standard Wave-Lengths from Xenon

Bureau of Standards[36]	
4500.978	4697.020
4524.680	4734.154
4582.746	4807.019
4603.028	4829.705
4624.275	4844.333
4671.225	4923.246

The elegance and refinement of interferometer methods for wave-length comparisons are truly remarkable, and most of the above values may be considered to be correct to 1 part in several

[34] Meggers, To be published in B. S. Bulletin.
[35] Comptes Rendus 156, p. 945; 1913.
[36] Meggers, To be published in B. S. Bulletin.

millions. This is evidenced by the agreement between entirely independent determinations and also by the frequency differences of many of the rare gas spectral lines belonging to combination series. There are a few cases where differences of 0.004 A or more exist and the reasons for these are not often apparent. More work of this kind on the part of different observers is very much to be desired so that an extended system of accurate secondary standards will ultimately be produced. The principal sources of error which require attention in such investigations lie in (1) determining the order of interference, (2) correcting for the dispersion of phase change on reflection,[37] (3) maladjustments of the optical system and spectrograph, (4) operating conditions of the light sources, (5) attempts to measure close or complex lines, and (6) in the corrections to wave-lengths measured in air under other than normal conditions.

4. CORRECTIONS TO WAVE LENGTHS MEASURED IN AIR

The international wave-length standards are specified in dry air at 15° C and 760 mm of mercury pressure. Variations in the density or index of refraction of the air appreciably affect the absolute values of the wave-lengths and when such measurements are made in air whose temperature is not 15° C, or whose pressure is not equal to 760 mm, corrections must be applied to reduce them to their values under normal conditions. Furthermore, wave lengths measured in air can be changed to their values in a vacuum only by an exact knowledge of the indices of refraction of the air for these wave-lengths.

The Rowland system of wave-lengths was defined in air at 20° C and 760 mm and correction tables to reduce such wave-lengths or oscillation frequencies to vacuum values were used as given in Kayser's Handbuch 2, p. 514. Since the new International System specifying wave-lengths at 15° C has come into general use, there has been no uniformity in such corrections. The resulting confusion has been pointed out by Birge[38] who

[37] Meggers, B. S., Bulletin 14, p. 2; 1915.
[38] Astroph. J., 50, p. 72; 1918.

recommends that the recent work of the Bureau of Standards[39] on the index of refraction of air for wave-lengths from 2218 A to 9000 A be referred to for these corrections in all future work. The correction tables prepared by the Bureau of Standards are based upon some 1200 measurements of the index of refraction of air (0°, 15° and 30°C) for wave-lengths from the extreme ultra-violet, through the visible spectrum, and in the infra-red. International adoption and use of these tables will exclude all further possibility of ambiguity or question about the conversion of wave-lengths to standard air condition or to vacuum when such wave-lengths are not actually observed under these conditions.

BUREAU OF STANDARDS
March 25, 1921

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[39] B. S. Bulletin 14, p. 697; 1918.

AN UNFAMILIAR ANOMALY OF VISION AND ITS RELATION TO CERTAIN OPTICAL INSTRUMENTS

BY

W. B. RAYTON

Any optical instrument provided with a prism erecting system should satisfy the condition of complete erection of the image, that is, vertical and horizontal lines of the object should appear vertical and horizontal in the image within certain small limits. One one occasion the writer attempted to judge the character of this adjustment of a field glass by employing a well known method often used for making an approximate determination of magnifying power, viz., the method of simultaneous observation of an object by one eye through the telescope and by the other without the telescope. Choosing in this case a flag staff as the most suitable object, the two images did not appear parallel and the glass was condemned as imperfectly adjusted. Subsequent examination with the apparatus usually employed for the purpose proved that the adjustment was satisfactory. Curiosity aroused, the former method was then applied to a telescope without a prism erecting system and again the two images were found to be inclined to each other. Tests were next made on nearly all of the Scientific Staff of the Bausch & Lomb Optical Company and without exception all observed the same effect.

Reference to the literature disclosed the fact that the phenomenon is well known to physiological opticians but the knowledge seems not to have spread far out of their ranks. The subject was very thoroughly investigated by A. W. Volkman in 1864, whose measurements are extensively quoted by Helmholtz in his Handbook of Physiological Optics.

It appears, according to Volkman, Helmholtz and other more recent writers, that a horizontal line appears horizontal to both eyes in the great majority of cases but that a vertical line does

not in general appear vertical to either eye. In single binocular vision the two images are fused into a single vertical line but if the fusion be destroyed by any means so that the two images are seen separately they appear to converge downwards or upwards according as the convergence of the eyes is less or more than is required for fusion. Consideration is limited in this discussion to subjects who are not afflicted with pathologic cyclophoria nor with extraordinary degrees of astigmia.

In addition to the method described of looking at a vertical line through a telescope with one eye and alongside the telescope with the other, there are other methods of producing the two separate images necessary for the observation. One such method consists in inserting a prism before one eye with base in, that is, with the base turned toward the nose. If placed before the eye with base out the eyes will generally converge to accomplish fusion of the images unless the deviation of the prism be so great as to make observation difficult for other reasons. If instead of a single prism a bi-prism be placed before the one eye with edge vertical, three images of a single vertical line will be seen which cannot all be fused into a single one. Again, holding a pencil vertically in the line of sight look at the wall beyond it. Two images of the pencil will be seen. Under these circumstances the two images of the pencil will appear to diverge upwards.

It is significant that the amount of the divergence is generally about equal to the angle subtended at the gound by the interpupillary distance. It is equally significant that if, in the experiment with the pencil, instead of holding it vertical the top is tipped away from the observer so that it lies approximately at the inclination of a book in reading, the two images appear parallel. These facts lead to the conclusion that the phenomenon may be due to some effect of experience.

Now if the effect is due to some process of education it seems that children, whose experience in life has not been so extensive, should not exhibit it or if they do that it should be to a less degree. Six children were tested using the telescope and the pencil tests as described. Five, ranging from nine to twelve years of age, were certain they saw the two images as parallel. One girl of thirteen

was uncertain thinking that at times she saw them parallel and at other times as diverging slightly. It is recognized that in a purely qualitative test, such as this, consideration must be given to the lack of training of children's powers of observation. In order to test to some degree the value of their observation, however several pairs of lines were drawn, each pair on a separate sheet of paper, some pairs being parallel while others were of varying degrees of divergence. The children tested had no hesitation in detecting divergence of considerably less than 1° which is less than half the usual divergence seen by an adult. More precise tests on a greater number of subjects are contemplated. The experiment, incomplete as it is, tends to establish the truth of the education hypothesis.

As to the nature of the product of this hypothetical educational process, it must be one of two things; either, as suggested by Stevens, there comes to be established a slight cyclophoria which is manifest when the eyes are at rest but which is overcome whenever single binocular vision is obtained, or else it is purely a matter of interpretation.

Most of our work is done at comparatively short ranges and calls for continued effort on the part of the muscles which produce convergence. So much is this so that most of us, while possessing strong power of convergence, have practically lost the power to diverge the lines of sight. Very young children and savages possess both to about equal degree. It is worth noting in this connection that single binocular vision is one of the latest developments in the process of evolution. The prolonged tension to which the muscles of the eye are subjected possibly results in a position of the eyeball which, while abnormal with respect to condition at birth, let us say, is normal in the sense that it is peculiar to practically all civilized adults. The cyclophoria hypothesis may find support in the fact that the phenomenon under discussion varies in its character and amount if the head is tipped forward and the eyeballs rolled upward. A position of the head can generally be found which makes the images parallel but that position is not by any means the natural one.

On the other hand, there are two experiments which seem to cast doubt upon the validity of the cyclophoria hypothesis. If a distant vertical line be regarded with both eyes and the head in normal position, of course a single line is seen. Now by quickly interposing a prism, of about four prism dioptries preferably, with its base turned toward the nose, a double line is immediately seen, the lines being inclined to each other. If fusion can be accomplished they merge into a single line. If by voluntary effort the eyes can be held in their original position and the prism be suddenly withdrawn the line immediately appears single. On the basis of the cyclophoria hypothesis it seems necessary to assume that the introduction of the prism and the consequent destruction of fusion permits the eyes to assume a position of rest and that to get single binocular vision again it must be assumed that they rotate slightly on their anterior-posterior axes. With full recognition of the possibilities of motor responses to stimuli which have not yet reached the brain it seems impossible to believe that the brain does not ultimately recognize the stimulus especially if watching for it. It seems reasonable, therefore, in this experiment, to expect that the two lines first seen when the prism is interposed before the eye would appear parallel and that they would ultimately take a position of upward divergence. Conversely, when the prism is suddenly removed from the eye we should expect to see two crossed lines which by rotation about their point of intersection ultimately fused into one. Furthermore, it does not appear entirely unreasonable to expect that the mutual rotation of the image could be detected although this point is more open to question. A little practice is required before the image can be seen without momentary confusion, but after a little experience the single or the double image can be instantly recognized and the change from one to the other appears to occur instantaneously.

Again, regard two vertical parallel lines; hold a pencil vertical before the eyes and adjust distances so that the pencil appears tangent to one of the lines when seen by the right eye alone and tangent to the other line when seen by the left eye alone. Now fix on the plane of the two parallel vertical lines. The two parallel

lines appear parallel but the two pencil images appear to diverge upwards. Inasmuch as the lines on the back-ground appear single, vertical, and parallel, it must be assumed that the eyes have assumed the hypothecated position for single binocular vision, and if the phenomenon with which we are concerned is due to a departure of the eyes from this position when the required stimulus is absent, that we have now supplied the stimulus which should bring the eyes into working position. Under these conditions the two pencil images should appear to be parallel, but they do not and it seems impossible, therefore, that the phenomenon can be ascribed to cyclophoria. It seems much easier to believe that it is a matter of interpretation in the same class, possibly, as the interpretation of the inverted images of our retinas.

The phenomenon has a direct bearing on all optical instruments which employ a prism system for erecting the image such as prism binoculars, binocular microscopes, stereoscopic range finders, etc. In any of these instruments, unless single binocular vision is obtained, there is the impression that the instrument is out of adjustment on account of this apparent divergence of lines which should appear parallel. Furthermore, it is quite unsafe to attempt to discover by means of the expedient described above whether the image formed by a prism telescope is completely erected.

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SOFT X-RAYS FROM ARCS IN VAPORS*

BY
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The present paper is a preliminary report of a study of the potentials required to excite radiations in the frequency range between the regions accessible to the vacuum grating spectroscope and the long wave-length limit of the x-ray crystal spectrometer. This region is of the utmost importance in the development of a theory of atomic structure, for it is in this range that the transitions from x-ray to ordinary arc and spark spectra take place. We will use the word x-ray to denote all radiation from the inner part of the atoms as distinguished from that due to the displacement or removal of outer valence electrons alone.

For this portion of the spectrum the photo-electric effect of the radiation offers about the only means of observation. The method of measuring the potential required to excite radiation by observing its photo-electric effect has been previously applied in studying radiating potentials below the first ionization point. The apparatus used in the search for higher critical potentials is similar to that employed for the determinations of ionization and resonance potentials.[1]

The element studied is boiled in a vacuum tube containing cylindrical electrodes having the following arrangement. The hot wire cathode at the center is surrounded by a small grid of fine wire gauze. A second gauze grid is at a relatively large distance from this and a cylindrical-plate collecting-electrode outside and close to the outer grid. The inner grid is put at the variable potential $+V$, which maintains the arc. The outer grid at a potential $-V_1$ shields the collecting electrode from all electrons and negative ions. With the plate at a potential $+V_2$ greater than V positive ions also are kept away so that any current to the plate is due to photo-electrons emitted by the outer

* Published with the permission of the Director of the Bureau of Standards.

[1] Cf. for example, Bureau of Standards Scientific Papers Nos. 400 and 403.

grid when illuminated by the radiation from the arc. The ionization can be measured by making the collecting plate negative. Special care had to be taken with the electrical insulation on account of the high temperatures. The shielding of the plate from all ions both positive and negative can be tested readily and is an important precaution.

In obtaining readings the current from the cathode as well as the current to the collecting electrode was recorded for every setting of the voltage V. Results were plotted with current to electrode divided by the cathode current as ordinates and V as abscissae. It happens that the photo-electric effect of the radiation when so plotted gives a straight line relation with a change in slope at each critical potential. The physical significance of this very striking linear relation is not known, as yet. The value of such a relation in determining the critical points is, however, evident. Corrections for initial potential were made by comparing the observed ionization point with the value for this point known from our earlier experiments.

The following results have been obtained:

Potassium—Radiation curves show a sharp increase in radiation starting at 23 ± 1 volts. Curves from 23 volts to over 100 show a straight line relation. As the vapor pressure is increased a lower break in the curve at 20 volts, as well as 23 volts, is seen. Ionization curves under some conditions show a break at 23 volts but the latter measurement is not nearly as certain.

Sodium—A definite break in the radiation curve at 35 volts. The point is not nearly as sharp as with potassium. A break is also evident near 17 volts. As a trace of hydrogen is always present with sodium vapor this point may be due to hydrogen but consideration of the results with other elements suggests that it is a sodium point. Ionization curves likewise show a break in the curves at 35 volts.

Magnesium—A definite break in the radiation curves at 46 ± 1 volts. Evidence of a point near 34 volts. Curvature below this indicates other lower radiation potentials as is to be expected from the enhanced spectra. The data available were not sufficient to separate these points. Experimental difficulties due to high temperature are considerable with magnesium.

Phosphorus—A sharp break at 126 ± 1 volts. Definite breaks at 162 and near 99 volts. Evidence of lower points which were not measurable with any accuracy.

Sulphur—The radiation curves show breaks at 152 ± 5 and 121 ± 2 volts. The breaks became definite only at high vapor pressure under which conditions the currents were rather unsteady.

These various critical potentials in the range 20 to 160 volts correspond by the quantum relation to a wave length range from 600A to 80A.

In the case of the alkali metals with one valence electron it is evident that critical potentials above the first ionization potential must be due to ejection of electrons from the x-ray rings. The potential required to eject the electron is determined by the limiting frequency of an x-ray series and this potential is then both an ionization and radiation point.

In the case of polyvalent atoms we may expect to find potentials above the first ionization giving multiple ionization, as well as those giving x-radiation. Thus in the case of magnesium we can compute from spectral data that the potential required to eject both valence electrons is 22.6 volts, while 14.95 volts is required to eject the second electron from an ionized atom. As noted before, the present measurements were not sensitive enough to show these latter points clearly.[2] Nothing is known of the enhanced spectral series of the non-metallic elements here considered, so that there are no means for computing the values of the multiple ionization potentials.

From x-ray data on the K series, it is possible to compute the limiting frequencies of the L series down to atomic number 12 from the relation:[3]

$$La_1 = Ka - K\alpha_1$$

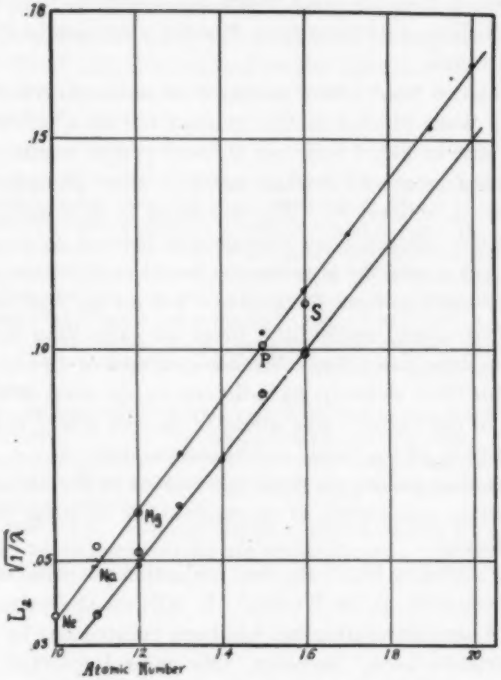
La_1 is a limiting frequency of an L series and gives the potential required to eject an electron from this L ring. In Fig. 1 the upper line shows the Mosely relation between the square root of the frequency and atomic number for La_1. The dots are points

[2] The spectral changes at these potentials will be discussed in detail in a paper by Foote, Meggers, and Mohler, now in preparation.

[3] Duane and Shimizu, Phys. Rev. 14, p. 67; 1919.

computed from the above relation. The circles are computed from the observed potentials; sodium 35, magnesium 46, phosphorus 126, sulphur 152. The sulphur point was known to be inaccurate. The phosphorus point falls on the straight line, not on the computed point. With magnesium the agreement is exact. However, the sodium point falls 5 volts above the extrapolated

FIG. 1



L—Limits for Low Atomic Numbers

line and the difference in this case is far beyond any probable experimental error.

Since obtaining this result, Professor Millikan has announced that he has found two isolated lines in the extreme ultraviolet spectrum of sodium at $\lambda = 372$ and 376A. These are undoubtedly

the $L\alpha$ lines. They correspond to a potential of 33 volts. Consideration of the accepted model of the sodium atom shows that these lines must be due to the valence electron falling into the L ring, with a loss in energy equivalent to 33 volts. The potential required to eject an L electron and excite these lines must then exceed the potential corresponding to the emission lines by the ionization potential, or 5 volts. The potential corresponding to the L limit is then 38 volts, while we observed 35. There is evidently a sharp departure from Moseley's relation at the lower end of the L line.

The combined results show also another series of critical potentials which when plotted in this manner fall on a straight line. The lower line in Fig. 1 is drawn through points computed from the following observed values: sulphur 121, phosphorus 99, magnesium 34, sodium 17. The last point is of doubtful origin and all except sulphur were inaccurately located on account of faintness, but a straight line relation seems to fit. It is possible to compute this line from x-ray data. For all the light elements and for these alone, apparently, there are faint lines $K\alpha_3$, $K\alpha_4$, $K\alpha_5$ and in some cases $K\alpha_6$. We have computed the frequency $K\alpha - K\alpha_5$ for these elements as indicated by the solid dots on the lower line of the figure. $K\alpha_6$ would fit as well where it is given but the terms α_3 and α_4 would not be satisfactory.

The potassium points are doubtless related to the M series but it is impossible to compute or extrapolate the M series to potassium accurately.

We have added to Fig. 1 the first ionization potential of Neon, atomic number 10, at 16.7 volts.[4] It falls on the extrapolated $L\alpha_1$ line. A similar relationship has been pointed out by Kossel. In Sommerfeld's book,[5] however, this value is plotted on the $L\alpha$ straight line, whereas probably the value for the lowest resonance potential should be used, as the ionization potential should correspond to an x-ray limit as given by Fig. 1.

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[4] Horton and Davies, Proc. Roy. Soc., 98 p. 124, 1920.

[5] Sommerfeld. Atombau und Spektrallinien. Second Edition, p. 548.

The agreement between computed and observed points on the upper line of Fig. 1 indicates that this new method of observing the limiting frequency of x-ray series is well justified. The accuracy attained in the case of phosphorus for example is about one per cent. The data for the lower line of Fig. 1 are much less satisfactory and further work must be done in this regard. This is a new x-ray series or group of series which, judging from the K-lines, only extends through a small range of light elements. The authors hope to apply this method in tracing out the beginnings of the M and N series in a similar manner.[6]

BUREAU OF STANDARDS, APRIL 27, 1921
WASHINGTON, D. C.

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[6] Recently Kurth has reported before the Physical Society results of experiments on characteristic X-rays *from solids*, by a photo-electric method. (Abstract Phys. R. 17 p. 528, 1921. Also Washington meeting of Am. Phys. Soc.) The same type of experiment was also carried out by us independently, with results which in some cases confirm the beautiful data of Mr. Kurth, and which will be extended and published later. The present work was undertaken to supplement this by using metals which are easily vaporized and also gases and non-metallic vapors. This is the first example of the excitation of x-rays in vapors. Many other observers have applied the photo-electric method in a search for low-voltage characteristic radiation from solids but without definite results.

Millikan has been able to identify many x-ray lines of wave-length longer than 130 Å in "hot spark" spectra. His results in sodium were noted above. This method of photographing the lines directly over this narrow spectral range and the extreme end of the x-ray series greatly adds to the value of determinations in which the critical voltages have been observed.

Note added Aug 8. Additional data are as follows: Chlorine 198 and 157 volts; Carbon 272 volts; Nitrogen 374 volts; and Oxygen 478 volts. The last three values are K limits.

NOTE ON MR. T. SMITH'S METHOD OF TRACING RAYS THROUGH AN OPTICAL SYSTEM

BY
C. W. WOODWORTH

Professor Southall has commented under the above title[1] upon the new English method of lens calculation, raising the doubt, however, whether there was actually any shortening of the process, and suggesting the comparison of a similar method based on the use of central perpendiculars. A still better method in my opinion is the use of lines perpendicular to the axis as proposed by me in a previous number of this *Journal*[2] applicable for general use but urged particularly for the purpose of avoiding serious inaccuracies of other methods in the special case of a ray with a very narrow θ angle passing from a curved to a flat surface.

The use of such perpendiculars does indeed simplify Smith's equation but no more so than the direct calculation of the foci based on the equation

$$n'c' \sin\theta' = n\, c \sin\theta$$

which expands at once into

$$\frac{n\,c}{n'c'} = \cos a' \cos a + \sin a' \sin a + \frac{\cos\theta}{\sin\theta}(\sin a' \cos a - \cos a' \sin a).$$

The equation by the other method using the parameter b normal to the axis can likewise be obtained in a similar form by expanding the equation

$$n'b' \cos\theta' = n\, b \cos\theta$$

which becomes

$$\frac{n\,b}{n'b'} = \cos a' \cos a + \sin a' \sin a - \frac{\sin\theta}{\cos\theta}(\sin a' \cos a - \cos a' \sin a)$$

After obtaining the value of c′ or of b′ by the second of one of these pairs of equations one can proceed to obtain the function of the θ' angle by the corresponding first equation.

The relation between the a and θ functions is shown by the equation

$$r \sin a = -c \sin\theta = b \cos\theta$$

[1] JOURNAL OF THE OPTICAL SOCIETY OF AMERICA. Vol. 5, pp. 14–21.
[2] *Ibid.*, Vol. 4, pp. 286–293.

For flat surfaces b and b′ are identical since the parameter must be measured along the surface; and since $\alpha = \theta$,

$$n' \sin \theta' = n \sin \theta.$$

In the case of paraxial rays through flat surfaces the angular functions disappear and

$$n'/u' = n/u.$$

The adoption of Smith's method at least when modified as above certainly results in a material reduction of the labor of calculating through a lens system. Even in theoretical work where one may desire greater accuracy than afforded by the four place sine-cosine table one can obtain the desired functions on the calculating machine by the equation $\cos \theta = \sqrt{1 - \sin^2 \theta}$ with a facility that I think will compete favorably with any other method of equal accuracy. It may be observed that it is not at all necessary actually to determine the successive foci or parameters but only the functions of the successive angles, thus eliminating two steps in the calculation of each intermediate surface.

UNIVERSITY OF CALIFORNIA

THE BLACKENING OF A PHOTOGRAPHIC PLATE AS A FUNCTION OF INTENSITY OF MONOCHROMATIC LIGHT AND TIME OF EXPOSURE, ON THE BASIS OF HURTER AND DRIFFIELD'S AND ROSS' FORMULAE

BY
P. S. HELMICK

In a previous contribution,[1] making use of a modification of Hurter and Driffield's formula,[2] a method was given of expressing the density of a photographic plate exposed to monochromatic light as a function of intensity of light and time of exposure to light. This expression readily lends itself to the calculation of the variation of Schwarzschild's exponent p[3] with intensity of light.

It has also been pointed out,[4] that this method gives results which fit the facts fairly well for densities of plates between 0.2 and 5. For densities less than 0.2, the method gives values which are greater than the experimental values.

By making use of other density expressions, it is possible to obtain agreement between observed and computed values of blackenings within the limits of experimental error, and to compute with the same ease as before, the variation of Schwarzschild's exponent p with intensity of light.

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[1] Phys. Rev., 17, 135; 1921.

[2] Journ. Soc. Chem. nd., 9, 455; 1890.
The particular expression used was:

$$D = \frac{1}{a}\log_e\left[b-(b-1)e^{-t10^{A+B\log_{10}I+C\log^2 I}}\right],$$

where D is the density $\left(\text{density equals } \log_{10}\frac{\text{Light incident on plate}}{\text{Light transmitted by plate}}\right)$ of the plate exposed for a length of time t to a monochromatic light of intensity I. a, b, A, B, and C are parameters dependent upon the wave-length of light and the characteristics of the plate.

[3] Astrophys. Journ., 11, 89; 1900. The expression states that for equal blackenings, $It^p = \text{constant}$.

[4] Phys. Rev., 17, 411; 1921.

In order to contrast these several methods of expressing the blackening of a photographic plate exposed to various intensities of monochromatic light for different times of exposure, a set of experimental data as previously obtained[5] for a Seed 23 plate exposed to monochromatic light of wave-length 550$\mu\,\mu$ for lengths of time from 1 second to 15 minutes will be utilized, and the blackening equations found according to the four following modifications of Hurter and Driffield's[6] or Ross'[7] equations:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-10^A I^{B+C\log_{10} I_t}}\right] \quad \ldots\ldots (1)$$

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^{B+C\log_{10} I_t})^{\frac{1}{k}}}\right] \quad \ldots\ldots (2)$$

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s 10^A I^{B+C\log_{10} I_t}}\right] \quad \ldots\ldots (3)$$

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^A I^{B+C\log_{10} I_t})^{\frac{1}{k}}}\right] \quad \ldots\ldots (4)$$

(1) and (2) reduce to Hurter and Driffield's equation, and (3) and (4) reduce to the equivalent of Ross' equation, upon setting $B=1$ and $C=0$ for (1) and (3), and setting $B=1$, $C=0$, and $k=1$ for (2) and (4).

THEOREM FOR DETERMINING COEFFICIENTS IN BLACKENING EQUATIONS BY SEMI-GRAPHICAL METHODS

An expression will be developed at this point, which will be of use in the semi-graphical determination of the parameters occurring in each of the equations.

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[5] *Loc. cit.*

[6] *Loc. cit.*

[7] JOURN. OPTICAL SOC. OF AMERICA, 4, 261; 1920.

Ross' original formula is:

$$D=d_m\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-kr^s It}\right],$$

where D is the density of a plate exposed for a time t to light of intensity I, d_m is the density for infinite exposure, k is the sensitivity factor, and r is the common ratio of the sensitivities of the various groups of grains in the plate.

Let the density D of a photographic plate exposed to monochromatic light of intensity I be expressed in the form:

$$D=\frac{1}{a}f(c\,t),$$

where t is the time of exposure, a is a constant of the plate, and c is some function of I and t.

Consider a blackening curve for a plate exposed to monochromatic light of constant intensity, plotted on tracing paper to a $\log_{10}D-\log_{10}t$ scale (cf. one of the curves of Fig. 3).

Suppose that it is possible to make this curve coincide with some curve $y=\Phi(x)$, orienting the curves so that the y and the $\log_{10}D$ axes, and the x and the $\log_{10}t$ axes are parallel.

Let l and m be the respective x and y intercepts of the $\log_{10}D$ and the $\log_{10}t$ axes.

Then

$$x=l+\log_{10}t,$$

and

$$y=m+\log_{10}D.$$

If x and y are so chosen that

$$y=\log_{10}aD,$$

and

$$x=k\log_{10}ct;$$

then

$$k\log_{10}ct=l+\log_{10}t,$$

and

$$\log_{10}aD=m+\log_{10}D$$

From whence follows that

$$c=10^{\frac{l}{k}\frac{1}{l}-1},$$

$$l=A+B\log_{10}I+C\log_{10}^2 I, \quad \ldots\ldots(5)$$

or if

$$c=10^{\frac{1}{k}(A+B\log_{10}I+C\log_{10}^2 I)}t^{\frac{1}{k}-1}, \quad \ldots\ldots(6)$$

or

$$c=(10^A I^{B+C\log_{10}I}t^{1-k})^{\frac{1}{k}}; \quad \ldots\ldots(6')$$

and as

$$a=10^m, \quad \ldots\ldots(7$$

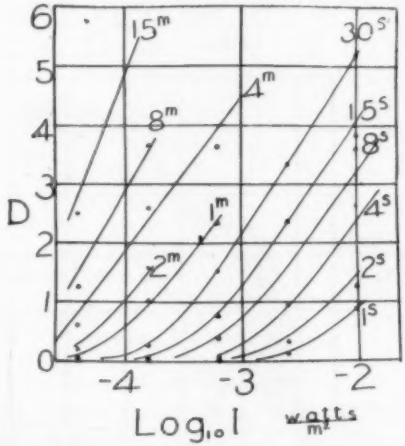
so

$$D=10^{-m}f(10^A I^{B+C\log_{10}I})^{\frac{1}{k}} \quad \ldots\ldots(8)$$

Thus if a curve $D = f(ct)$ can be found such that an experimental $\log_{10}D - \log_{10}t$ curve with intensity constant, fits it when the respective axes are parallel, m can be determined, and the l can be found for the particular intensity used to obtain the $\log_{10}D - \log_{10}t$ curve. Various l's can now be determined for different I's using other $\log_{10}D - \log_{10}t$ curves, and from this set of values of l; A, B, and C can be obtained. m, A, B, C, k, and the form of the f-function being known, by substitution in (8), the density of the plate is now expressible in terms of the intensity of the monochromatic light and the time of exposure.

This method will now be used to determine the constants of (1), (2), (3), and (4) from the data represented by the points of Fig. 1.

FIG. 1



Experimental data to be fitted by blackening curves. Points represent experimental data, curves are drawn free-hand through the points. Seed 23 Plate. $\lambda = 550\ \mu\mu$.

EQUATION 1

$$D = \frac{1}{a}\left[b - (b-1)e^{-10^{A I^{B + C\log_{10} I}} t}\right].$$

A method of obtaining the constants of this equation has already been described in a former paper (loc. cit.), but the process will again be outlined, in order that the results of this

equation may be contrasted with the results of the other equations.

In order to represent the data of Fig. 1 according to equation (1), set

$$c = 10^{A} I^{B+C \log_{10} I}.$$

(6) then becomes identical with this last expression if the k of (6′) is set equal to unity.

(1) will now reduce to

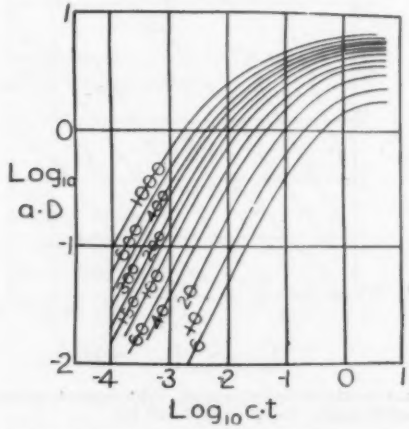
$$D = \frac{1}{a} \log_e \left[b - (b-1) e^{-ct} \right] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (9)$$

an expression of the form, $D = \frac{1}{a} f(ct)$,

where $f(z) = \log_e [b - (b-1)\, e^{-z}]$.

From (9), $\log_{10} aD$ is now plotted against $\log_{10} ct$, as in Fig. 2, with b a parameter,—values of b ranging from 6 to 1000. A

FIG. 2



Monographic chart for determination of parameters in the blackening equation:

$$D = \frac{1}{a} \log_e \left[b - (b-1) e^{-10^{A} I^{B+C \log_{10} I} t} \right].$$

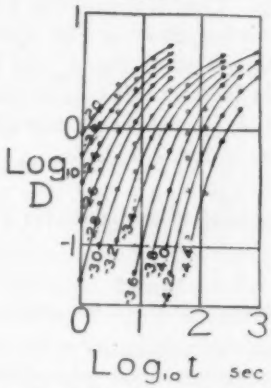
separate $\log_{10} aD$—$\log_{10} ct$ curve will, of course, be obtained for each distinct value of b. The next step is to obtain a set of curves in which the experimental data are plotted to a $\log_{10} D$—$\log_{10} t$

scale for various intensities of light,—one curve for each separate intensity chosen.

If the experimentally determined points of Fig. 1 are used as they stand, it will be possible to draw only five $\log_{10}D-\log_{10}t$ (intensity being constant) curves for this set of data, for but five different values of intensity were used.

From these five curves, a value of I can be obtained for each of the five intensities.

FIG. 3



Data from the curves of Fig. 1 plotted in a form suitable for the graphical determination of parameters in the four blackening equations.

From (5), five equations

$$I_1 = A + B\log_{10}I_1 + C\log_{10}{}^2I_1,$$

$$\cdots\cdots\cdots\cdots$$

$$I_5 = A + B\log_{10}I_5 + C\log_{10}{}^2I_5,$$

will then result, from which A, B, C, and I can be determined by the method of least squares.

If it is possible to obtain a much greater number of equations than five, the A, B, and C can be determined by a method of Campbell,[8] which is considerably simpler than the standard method of least squares.

Accordingly, free-hand curves representing various times of exposure from 1 second to 15 minutes were drawn in Fig. 1.

[8] Phil. Mag., 39, 177; 1920.

From the intersection of these curves with the $\log_{10}I$ ordinates $-2.0, -2.2, \ldots, -4.4$, Fig. 3 is obtained. This figure shows the relationship between $\log_{10}D$ and $\log_{10}l$ for thirteen different values of intensity.

The next step is to determine some value of b which gives the best fit to these thirteen curves, and also some constant value of $\log_{10}aD$ which gives an abscissa along which one can slide Fig. 3, keeping the $\log_{10}D=0$ intercept on the $\log_{10}aD$ axis constant. A trial shows that the curve $b=200$, and an intercept $m=-0.2$ gives the best fit. That is to say, Fig. 3 will be moved over Fig. 2, keeping the axis $\log_{10}D=0$ of Fig. 2 always a distance of 0.2 below the axis $\log_{10}aD=0$ of Fig. 3, and the values l of the $\log_{10}D$ intercept on the $\log_{10}aD$ axis will be found for each of the thirteen values of $\log_{10}I$, i.e., whenever any of the various $\log_{10}I$ equals constant curves: $-2.0, -2.2, \ldots, -2.4$, coincide with the curve $b=200$.

Performing this operation gives values for l as found in Table 1, Column l_1.

TABLE 1

$\text{Log}_{10}I$	Values of l for Equations (1), (2), (3), and (4)			
	l_1	l_2	l_3	l_4
−2.0	−2.36	−1.64	−0.30	0.06
−2.2	−2.53	−1.86	−0.50	−0.14
−2.4	−2.73	−2.02	−0.68	−0.30
−2.6	−2.92	−2.22	−0.88	−0.51
−2.8	−3.12	−2.42	−1.06	−0.72
−3.0	−3.36	−2.65	−1.30	−0.94
−3.2	−3.25	−2.84	−1.50	−1.14
−3.4	−3.76	−3.05	−1.70	−1.35
−3.6	−3.90	−3.24	−1.88	−1.52
−3.8	−4.08	−3.44	−2.06	−1.71
−4.0	−4.22	−3.60	−2.25	−1.87
−4.2	−4.42	−3.80	−2.47	−2.13
−4.4	−4.63	−4.02	−2.64	−2.32

From (5),

$$l=A+B\log_{10}I+C\log_{10}{}^2I,$$

so the following equations are now at hand for the determination of A, B, and C:

$$\begin{aligned}
-2.36 &= A - 2.0B + 4.00C,\\
-2.53 &= A - 2.2B + 4.84C,\\
-2.73 &= A - 2.4B + 5.76C, \qquad \text{(I)}\\
-2.92 &= A - 2.6B + 6.76C,\\
\\
-3.12 &= A - 2.8B + 7.84C,\\
-3.36 &= A - 3.0B + 9.00C,\\
-3.56 &= A - 3.2B + 10.24C, \qquad \text{(II)}\\
-3.76 &= A - 3.4B + 11.56C,\\
\\
-3.90 &= A - 3.6B + 12.96C,\\
-4.08 &= A - 3.8B + 14.44C,\\
-4.22 &= A - 4.0B + 16.00C, \qquad \text{(III)}\\
-4.42 &= A - 4.2B + 17.64C,\\
-4.63 &= A - 4.4B + 19.36C.
\end{aligned}$$

In determining approximate values of A, B, and C by Campbell's[9] "Method of the Zero Sum," the equations are divided into three sets, (I), (II), and (III), and the coefficients of each of the terms are added together, the theory assuming that with a *large* number of equations, the sum of the errors in each sum of coefficients approaches zero. With the small number of equations used here, it is justifiable to use this method as a rough approximation only.[10] Performing such additions, the following three "normal" equations result:

$$\begin{aligned}
-10.54 &= 4A - 9.2B + 21.36C,\\
-13.80 &= 4A - 12.4B + 38.64C,\\
-21.25 &= 5A - 20.0B + 80.40C.
\end{aligned}$$

From these equations, the values of the constants can be determined as follows:

$$\begin{aligned}
A &= 0.24 \pm 0.03,\\
B &= 1.42 \pm 0.03,\\
C &= 0.075 \pm 0.008.
\end{aligned}$$

As we have already found $b = 200$, and $m = -0.2$, and from (7) $\frac{1}{a} = 10^{-m}$, from (1),

$$D = 10^{0.2} \log_e \left[200 - (200-1)e^{-10^{0.24} I^{1.42 + 0.075 \log_{10} I}} \right].$$

[9] *Loc. cit.*

[10] Roeser's method of fitting curves to observations taken at equal intervals of the independent variable would be very applicable to these data. Its use is equivalent to

For the present purpose it is scarcely worth while to simplify this formula further, for the most direct way of calculating values of D for various values of I and t is to compute the value of l corresponding to any value of I from

$$l=0.24+1.42\log_{10}I+0.075\log_{10}^2 I,$$

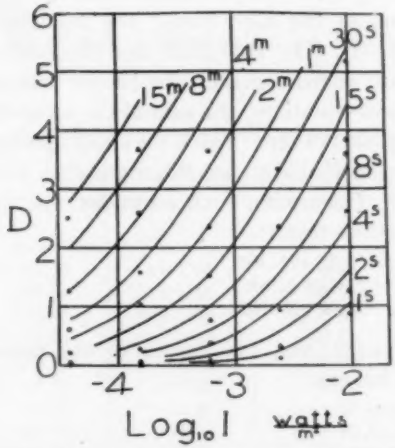
and then knowing l, b, a, and t, to read off the value of $\log_{10}D$ from Fig. 2.

In this manner the curves of Fig. 4 were obtained, the points representing the experimental values as used in Fig. 1.

EQUATION 2

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^B+C\log_{10}I)^{\frac{1}{k}}}\right].$$

FIG. 4



Agreement between experimental values (points) of the data of Fig. 1 and blackening curves computed from the expression:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-10^A I^B+C\log_{10}I}\right]$$

Note the poor agreement between observed and computed values for small densities.

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the ordinary method of least squares which assumes that the independent variable is not subject to error. Of course, this assumption would not be strictly true for these data. See Sci. Papers of Bur. Standards, No. 388; 1920 for details of this method.

From Fig. 4 it can be seen that the agreement between the experimental and the computed values of density is not good when small blackenings are considered. The experimental and the computed values of density can never agree for low blackenings, because for small times of exposure, the slopes of the curves in Figs. 2 and 3 are different, making coincidence of the curves impossible. Calculation and also actual measurement show that no matter what the value of b, the slopes of the curves in Fig. 2 are equal to unity when $t=0$, while the slopes of the experimental curves are equal to 1.67 for $t=0$.

These considerations lead to the addition of the exponent $\frac{1}{k}$ to (1), thus forming a new equation, (2), the function of the exponent being to increase the slope of the curves.

In order to reduce (2) to the form $D=\frac{1}{a}f(ct)$, set

$$c=(10^{A}I^{B+C\log_{10}I}t^{1-k}),$$

whence from (2),

$$D=\frac{1}{a}\log_{10}e\left[b-(b-1)e^{-ct}\right],$$

and so the intercepts l and m of the experimental $\log_{10}D-\log_{10}t$ curve on the $\log_{10}aD-k\log_{10}ct$ curve suffice to determine all of the constants.

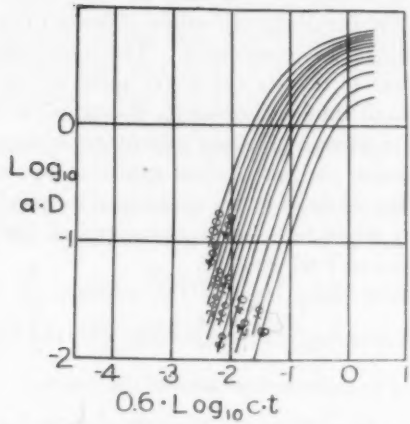
The $\log_{10}D-\log_{10}t$ curve is the one already described as Fig. 3.

It is necessary to plot a set of $\log_{10}aD-k\log_{10}ct$ curves for each different value of k which it is thought desirable to use. Fig. 5 shows such a set of curves plotted for $k=0.6$. It will be seen that the slopes of the experimental curves of Fig. 3. and the curves of Fig. 5 are the same for small values of t, so better agreement[11] may now be expected for small times of exposure.

Trial shows that the best fit between Figs. 3 and 5 is obtained when $b=1000$ and $m=0.08$. In order to find the various l's for the experimental curves of Fig. 3, Fig. 3 is superimposed on Fig. 5, so that the axis $\log_{10}D=0$ is always a distance 0.08 above the axis $\log_{10}aD=0$. Fig. 3 is then shifted along Fig. 5, preserving

[11] $1/0.6=1.67$, which is the slope of the curves of Fig. 3.

FIG. 5

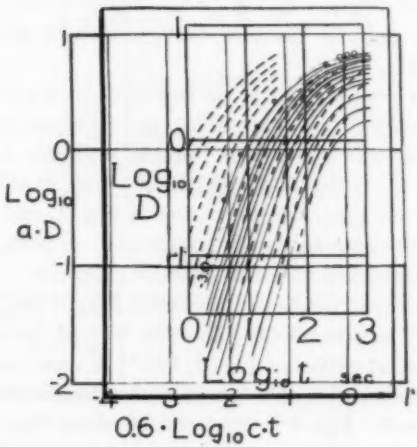


Monographic chart for determination of parameters in the blackening equation:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^B+C\log_{10} I\ i)^{\frac{1}{k}}}\right].$$

$k=0.6$

FIG. 6



Method of obtaining parameters of curve −3.0 of Fig. 3 by superposition of Fig. 3 on Fig. 5. $b=1000$. x and y intercepts give $l=-2.65$, $m=0.08$.

the same distance between the axes, and whenever a curve of Fig. 3 coincides with the curve $b=1000$ of Fig. 5, the l will be read off as the distance between the axis $\log_{10}l=0$ and the axis $\log_{10}ct=0.6$.

Fig. 6 is an attempted representation of this process. It shows the two curves placed so the $\log_{10}D=0$ axis is a distance 0.08 higher than the $\log_{10}aD$ axis, and pictures a sample "coincidence" of the curve $\log_{10}I=-3.0$ of Fig. 3 with the curve $b=1000$ of Fig. 5. The circles and the heavy line marked *−3.0* is the curve $\log_{10}I=-3.0$, and the faint line marked *1000* cutting the line $\log_{10}I=-0.3$ is the line $b=1000$. This illustration is chosen to show what is probably the worst fit in the set. The value of l for this particular curve $\log_{10}I=-3.0$, or the distance between the axes $\log_{10}D=0$ and $\log_{10}aD=0$, is -2.65.

Proceeding in this way, the various values of l corresponding to the thirteen different curves $\log_{10}I=-2.0$, etc. of Fig. 3 are found as set down in the column l_2 of Table 1. A, B, and C can now be determined by the "Method of the Zero Sum," as given under Equation 1.

The "normal" equations resulting from a division of the equations $l=A+B\log_{10}I+C\log_{10}{}^2I$ into the three groups I, II, and III as before, are:

$$\begin{aligned} -7.74&=4A-9.2B+21.36C,\\ -10.96&=4A-12.4B+38.64C,\\ -18.10&=5A-20.0B+80.40C. \end{aligned}$$

A solution of these equations gives:

$$\begin{aligned} A&=0.50\pm0.02,\\ B&=1.09\pm0.02,\\ C&=0.016\pm0.005. \end{aligned}$$

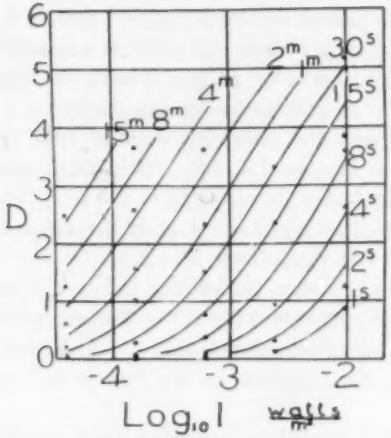
The blackening equation is accordingly deduced by simplifying the expression:

$$D=10^{-0.08\log_e}\left[1000-(1000-1)e-(10^{0.50}I^{1.09+0.016\log_e I_l})^{\frac{1}{0.6}}\right].$$

This equation, by a semi-graphical solution similar to that described under Equation 1, gives a representation of the data as shown in Fig. 7, the curves giving the computed values by the formula, and the points the experimental values as before.

The accuracy of fit is within the limit of experimental error of the data.

FIG. 7



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D=\frac{1}{a}\log_e\left[b-(b-1)e^{-(10^A I^{B+C\log_{10}I})^{\frac{1}{k}}}\right].$$

EQUATION 3

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1}e^{-r^s 10^{A I^{B+C\log_{10}I}}}\right].$$

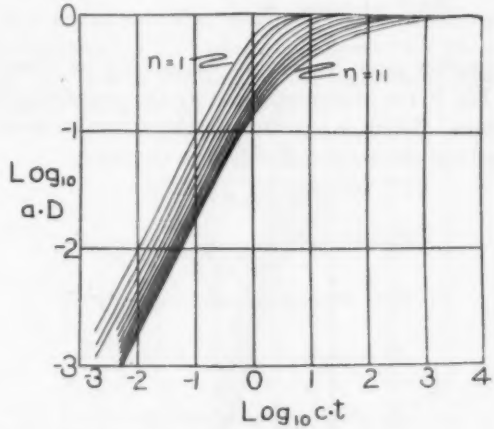
(3) is transformed into the desired form $D=\frac{1}{a}f(ct)$ by putting $c=10^{A I^{B+C\log_{10}I}}$ and as c is then equivalent to (6′) where the k of (6′) is set equal to unity, a, A, B, and C can be obtained from the values of the intercepts l and m by (5) and (7), and u and r can be determined by the particular curve of the $D=\frac{1}{a}f(ct)$ family which gives the best fit to the experimental $\log_{10}D-\log_{10}I$ curves.

The family of curves to be plotted on $\log_{10}aD - \log_{10}ct$ co-ordinates is given by

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^2ct}\right].$$

Fig. 8 shows a family of eleven such curves[12] plotted, with the parameter n having eleven different values as shown, — $n=1$, $n=2$, $n=3$, . . . , $n=11$, with r having the value $\frac{1}{2}$.

FIG. 8



Monographic chart for determination of parameters in the blackening equation:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^2 10^{A\,I^{B+C\log_{10}I}}}\right].$$

An attempt to fit the curves of Fig. 3 to those of Fig. 8 shows that the values $m=-1.05$, and $n=11$ give the best results.

By shifting the curves over each other, always keeping the axis $\log_{10}D=0$ on the line $\log_{10}aD=-1.05$, values of l are found as given in Table 1, Column l_3.

---

[12] Cf. Ross., Fig. 4, *loc. cit.*

The "normal" equations for the determination of A, B, and C, are accordingly:

$$\begin{aligned} -2.36 &= 4A - 9.2B + 21.36C, \\ -5.56 &= 4A - 12.4B + 38.64C, \\ -11.30 &= 5A - 20.0B + 80.40C, \end{aligned}$$

from whence

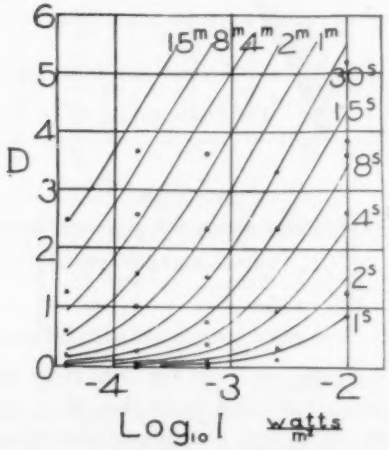
$$\begin{aligned} A &= 1.85 \pm 0.02, \\ B &= 1.10 \pm 0.02, \\ C &= 0.019 \pm 0.004, \end{aligned}$$

and so the equation of blackening is found to be:

$$D = 10^{1.05}\left[1 - \frac{1}{11}\sum_{s=0}^{s=10} e^{-(\frac{1}{2})^s (10^{1.85} I^{1.10} + 0.019 \log_{10} I)}\right].$$

This equation gives values for blackening as shown in the curves of Fig. 9, the points representing the experimentally determined values. There is a noticeable departure between the calculated and the observed values for low densities.

FIG. 9



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D = \frac{1}{a}\left[1 - \frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s 10^{A I^{B+C \log_{10} I}}}\right].$$

Note the poor agreement between observed and computed values for small densities.

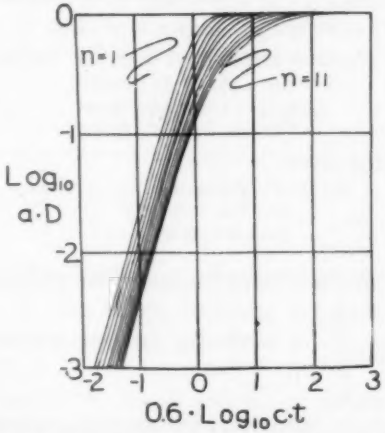
EQUATION 4

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^4 I^B+C\log_{10} I\, i)^{\frac{1}{k}}}\right].$$

After having observed the poor fit of Equation 3 for low intensities, it is natural to try to modify the equation to obtain a more accurate representation of the data.

In the curves of Fig. 8, the parameter r was set equal to $\frac{1}{2}$, if it were given some other value,[13] the shape of the curves would be altered. For small blackenings, the slope of the theoretical curves in Fig. 8 is equal to unity, while the slope of the experimental curves is equal to 1.67. Calculation shows that no matter what the values of r or of n, the slope of the theoretical curves of Fig. 8 will be always equal to unity for $t=0$.

FIG. 10



Monographic chart for determination of parameters in the blackening equation:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1} e^{-r^s(10^4 I^B+C\log_{10} I\, i)^{\frac{1}{k}}}\right].$$

$k=0.6$

[13] Ross states that the field of practical emulsions should be well covered with $r=\frac{1}{2}$, and assigning values of n from 1 to 10.

Hence an accurate expression for small blackenings becomes impossible with Equation 3, but a suitable modification,—or Equation 4, will give the same slope to each set of curves.

To convert (4) into the form $D=\frac{1}{a}f(ct)$, put

$$C=(10^{A}I^{B+C\log_{10}I}I^{1-k})^{\frac{1}{k}},$$

an expression which is equivalent to (6′).

D then becomes:

$$D=\frac{1}{a}\left[1-\sum_{s=0}^{s=n-1}\frac{-r^{s}ct}{e}\right],$$

and so a family of these curves with $r=\frac{1}{2}$ and n ranging from 1 to 11 is plotted on a $\log_{10}aD-k\log_{10}ct$ scale, as in Fig. 10. The best fit is given by the curve $n=11$, with $a=-0.75$.

Placing the $\log_{10}D=0$ axis of Fig. 3 on the line $\log_{10}aD=-0.75$ of Fig. 10, and sliding Fig. 3 over Fig. 10 in a horizontal direction gives values of I as shown in Column I_4 of Table 3.

A, B, and C are then determined from the "normal" equations:

$$\begin{aligned}-1.01&=4A-\ 9.2B+21.36C\\-4.15&=4A-12.4B+38.64C\\-9.55&=5A-20.0B+80.40C,\end{aligned}$$

and their solution gives:

$$\begin{aligned}A&=\ 2.05\pm\ 0.03\\B&=\ 1.02\pm\ 0.03\\C&=0.007\pm0.007.\end{aligned}$$

The expression for blackening according to Equation 4 is as a consequence:

$$D=10^{0.75}\left[1-\frac{1}{11}\sum_{s=0}^{s=10}e^{-(\frac{1}{2})^{s}(10^{2.05}I^{\,1.02+0.07\log_{10}I})^{\frac{1}{0.6}}}\right].$$

Fig. 11 shows the agreement between the experimental values (points) and the computed values (curves).

SCHWARZSCHILD'S EXPONENT p

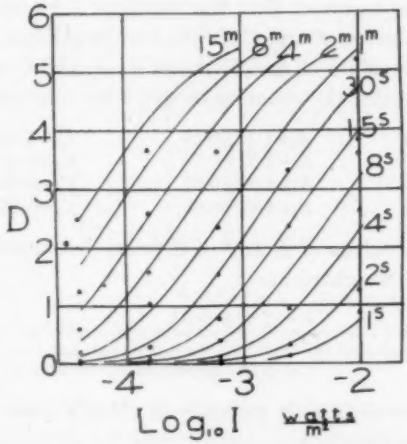
Schwarzschild[14] states that It^{p} must be constant for equal blackenings. The form of equations (1), (2), (3), and (4) shows that for

[14] *Loc. cit.*

equal blackenings $I^{B+C\log_{10}I}t$ must be constant, or $It^{(B+C\log_{10}I)^{-1}}$ must be constant.

Hence the expression[15] $(B+C\log_{10}I)^{-1}$ is the equivalent of Schwarzschild's p.

FIG. 11



Agreement between experimental data of Fig. 1 (points) and blackening curves computed from:

$$D=\frac{1}{a}\left[1-\frac{1}{n}\sum_{s=0}^{s=n-1}e^{-r^s(10^A I^{B+C\log_{10}I}t)^{\frac{1}{k}}}\right]^{1}$$

Therefore one of the advantages of using equations (1), (2), (3), and (4), lies in the fact of the resulting simplicity attached to finding the value of p, and any variation[16] in the value of p with the intensity of light.

As $p=(B+C\log_{10}I)^{-1}$, the B and the C should be identical for each of the four equations (1), (2), (3), and (4). The values of B in the four equations were found to be: 1.42, 1.09, 1.10, and 1.02; and the values of C were found to be: 0.075, 0.016,

[15] If l be expressed in the form

$l=A+B\log_{10}I+C\log_{10}{}^2I+D\log_{10}{}^3I+\ldots\ldots\ldots\ldots+N\log_{10}{}^nI,$

then $p=B+C\log_{10}I+D\log_{10}{}^2I+E\log_{10}{}^3I+\ldots\ldots\ldots\ldots+N\log_{10}{}^nI.$

[16] On variations of p with intensity of light cf. Astrophys. Journ., 29, 154; 1909; and Ann. der Phys., 41, 751; 1913.

0.019, and 0.007. As (2) and (4) give better fits of experimental values to computed values than do (1) and (3), weights 2 and 1 may be respectively assigned. With these conservative values of weights, the most probable value of B is 1.12 ± 0.05, and the most probable value of C is 0.023 ± 0.012.

If it now be assumed that the numbers 1.12 and 0.023 give the correct values of B and C for the four equations, the following most probable values of the constants A, B, and C are found, the subscripts denoting the equation to which the constant is attached:

$A_1 = -0.17 \pm 0.04$	$B_1 = 1.12 \pm 0.04$	$C_1 = 0.023 \pm 0.009$
$A_2 = 0.51 \pm 0.03$	$B_2 = 1.12 \pm 0.03$	$C_2 = 0.023 \pm 0.007$
$A_3 = 1.86 \pm 0.02$	$B_3 = 1.12 \pm 0.02$	$C_3 = 0.023 \pm 0.005$
$A_4 = 2.22 \pm 0.03$	$B_4 = 1.12 \pm 0.03$	$C_4 = 0.023 \pm 0.008$

So the best values of B and of C from the equations (1), (2), (3), and (4) as weighted are:

$$B = 1.12 \pm 0.03,$$

and

$$C = 0.023 \pm 0.007.$$

With these values it is possible to calculate the variation of p when $\log_{10} I$ varies from -2.0 to -4.4

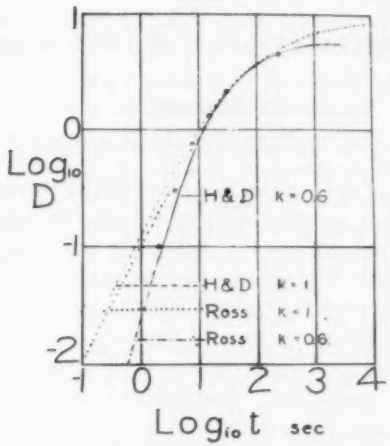
$$\text{For } \log_{10} I = -2.0,\ p = 0.93 \pm 0.03,$$
$$\text{For } \log_{10} I = -4.4,\ p = 0.98 \pm 0.04.$$

It can be calculated that the chances are 26 to 100 that the value of p remains constant when the intensity of light changes by a factor $1:10^{2.4}$, or 1:250, so it is probable that the p of Schwarzschild is not a constant in this case.

COMPARISON OF THE FOUR EQUATIONS

From the figures showing the fit of the empirical curves to the experimental data, it is easy to see that for the data at hand, equations (2) and (4) give the best fit, but accurate statistical methods must be applied before it can be said that one of these equations is superior to the other. The data given here are probably not accurate enough to be adjusted to a degree which would disclose which of the two equations gives the best fit.

FIG. 12



Curves showing agreement to be expected between experimental data (curve −3.0 of Fig. 3) and curves computed from modifications of Hurter and Driffield's and Ross' formulae. Circles represent experimental data, curves represent computed values.

Fig. 12 shows the fit of the four different equations to the experimental curve $\log_{10}D = -3.0$. The circles give the experimental values, and the curves of equations (1), (2), (3), and (4) are respectively labelled: H & D, k = 1; H & D, k = 0.6; Ross, k = 1; and Ross, k = 0.6.

PHYSICAL LABORATORY,
THE STATE UNIVERSITY OF IOWA,
April 13, 1921

A PORTABLE VACUUM THERMOPILE*

BY
W. W. COBLENTZ

I. INTRODUCTORY STATEMENT

The object of this paper is to describe a vacuum thermopile mounting which is rendered portable by attaching a calcium evacuator permanently to the radiometer container.

By this simple means one can maintain a vacuum without being obliged to keep the radiometer attached to a pump.

One of the chief complaints of workers in thermal radiometry is the uncertainty of the measurements, caused by unsteadiness of atmospheric conditions. Radiometers are necessarily of small heat capacity and, as a consequence, are greatly affected by air currents. The loss of heat by convection is very large. The sensitivity of a modern thermopile, (that is, its rise in temperature when exposed to radiation) is doubled and that of a thermocouple of fine (0.01 mm) wire is quadrupled by placing it in an evacuated receptacle.[1]

The container for a vacuum bolometer or thermopile differs from that of an evacuated, glass-bulb, incandescent lamp in that it cannot be thoroughly heated to expel the adsorbed water vapor and occluded gases. The radiometer container is usually provided with a fluorite or rock salt window which cannot be heated without risk of breakage, and hence is attached with a cement that may give off vapors. Furthermore the radiometer receiver is covered with lamp black, etc., containing absorbed gases which slowly escape into the evacuated container.

As a result of these difficulties attempts to seal off the vacuum radiometer permanently from the evacuating device have not been successful.[2] Consequently it is necessary to provide a vacuum pump or other means for removing the hydrocarbon vapors com-

* Published with the permission of the Director of the Bureau of Standards.

[1] B. S. Bulletin 11, pp. 132 and 613; 1914.

[2] Buchwald, Ann. der Phys., (4), 33, p. 928; 1910.

ing from the stop-cock grease and the gases disengaged from the walls or leaking through the joints of the vacuum chamber.

In a previous paper[3] a simple radiometric attachment was described for maintaining a vacuum by means of metallic calcium, which has the property of chemically combining with all atmospheric gases except argon. It is a simplification of the method used by Soddy,[4] in that the calcium is contained in a quartz tube which is heated to a low red by either a Bunsen burner, an alcohol lamp, or electrically. The calcium, being covered with a thin coating of oxide, does not combine with the quartz container unless it is heated to a very bright red color.

Portable vacuum thermopiles in which the vacuum is maintained by means of calcium have been in use by the writer for nearly seven years. One of the vacuum thermocouple containers (No. 7) used in stellar radiometry[5] was provided with a new quartz tube and calcium on March 22, 1919. On January 19, 1921 there was found only a slight decrease in vacuum, (total pressure perhaps 0.5 mm). This was easily remedied by warming the quartz tube containing the calcium. Container No. 6 used in stellar radiometry was heated Dec. 10, 1917, for the first time after August, 1914, and then again the second time on January 19, 1921, i.e., twice in seven years. This vacuum container which began to leak soon after it was finished, was easily repaired by applying a coat of shellac. However, as mentioned in the previous paper, sufficient air had entered to impart to the Geissler tube discharge the characteristic blue color of argon. In the most recent attempt to remove this residual gas (pressure less than 1 mm after standing three years) prolonged heating of the calcium evacuator had no effect in diminishing the argon.

The linear vacuum thermopile, to be described presently, has been in use for about four years. The first container in which it was mounted always leaked seriously. Nevertheless, after closing the stop-cock (in 1917) and sealing it with Chatterton wax, the vacuum was maintained by occasionally heating the calcium.

[3] B. S. Bulletin 11, p. 185; 1914.

[4] Soddy, Proc. Roy. Soc., 78, p. 429; 1907. Baly's Spectroscopy, new Ed., p. 433. Also magnesium is reported to be good for "vacuum cleanup."

[5] B. S. Bulletin, 11, p. 624, Fig. 3, 1914.

II. DESCRIPTION OF THE THERMOPILE

The thermopile is constructed of bismuth wire, 0.08 mm in diameter, (0.1 mm would be less fragile and hence adapted for general use) and silver wire, 0.031 mm in diameter. The individual receivers are of tin, 1.7 by 1.1 by 0.018 mm. The total length of the receiver of the completed instrument of 12 elements is 12.5 mm, and the reistance is 6.7 ohms. The bismuth wire is cut to the proper length (4 mm) and pressed flat between glass plates to increase thermal conduction where it is in contact with the receivers and thus facilitate the attainment of thermal equilibrium.[6] The base upon which the thermoelements are mounted may be of ivory or of red fiber (shown in Fig. 1) which has been boiled in paraffin to shrink it and to expel the air.

The rate of rise of emf of the thermopile, when exposed to radiation, is so rapid that a galvanometer swing of 2 seconds is not retarded. In fact, when using a galvanometer having a single swing of 2 seconds, the response of the thermopile is too rapid for making observations comfortably. For example, 90 per cent (that is 18 cm) of a total deflection of 20 cm is obtained in about 1 second.

III. DESCRIPTION OF THE VACUUM THERMOPILE CONTAINER

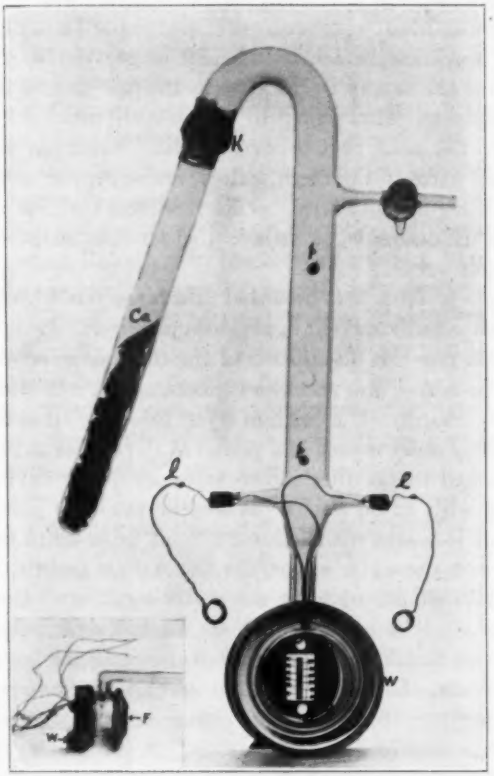
Radiometer containers which can be evacuated have been described in previous papers.[7]

The thermopile in its new vacuum mounting (20 cm in height) is shown in Fig. 1, which is a rear view. This mounting is in the form of a cup made out of one piece of glass, which is covered with a thick glass window, *W*. The inset, in the lower left-hand corner is a side view of a similar vacuum thermopile container used in stellar radiometry. The quartz tube, (standard size quartz test tube) containing the calcium *Ca*, is attached to the glass tube by means of Khotinsky cement, *K*. Pyrex glass tubing may be used instead of quartz and joined directly to the glass, thus

[6] B. S. Bulletin 11, pp. 132 and 613; 1914.
[7] B. S. Bulletin, 9, p. 39, 1911; 13, p. 444, 1916.

dispensing with the cement, but it is likely to collapse on accidental overheating.

FIG. 1



VACUUM THERMOPILE

The electrodes, p, of platinum wire 0.3 to 0.4 mm in thickness, are used for testing the evacuation. For this purpose an induction coil, or a 2,000 to 10,000 volt transformer is used.

The glass window, W, covering the rear of the container, is 7 mm thick. Experience shows that a thick plate of glass is

necessary in order to prevent warping by atmospheric pressure and conseqent leaking through the ground joint. The latter is closed with a combination of beeswax and tallow (stop-cock grease) and the edge is covered with Chatterton wax.

The fluorite window, covering the front of the container, is about 3 mm in thickness, which is sufficient to prevent warping when placed over the slot (8 by 16 mm) in the fine-ground face of the glass receptacle. It is joined to the glass by means of stop-cock grease and the outer edge is covered with Chatterton wax.

The lead wires to the thermopile, *l*, are of copper, sealed to the glass with a platinum wire. This refinement is not necessary, though it is desirable to reduce the number of places where leakage may occur.

The glass receptacle is mounted in a brass box which slides in vertical ways as illustrated in previous papers.[8]

In concluding this description of the thermopile container it is relevant to add a few remarks concerning the operation of the calcium evacuator. The calcium is out into small fragments with a chisel or by other means and placed in the quartz tube, which is then attached to the thermopile container. The device is then evacuated with an oil pump. While the pumping is in progress the calcium is heated with a Bunsen flame in order to cause it to combine with the water vapor, the residual air and the acetylene gas which is generated from the water vapor and the calcium carbide which is formed in previous use. Keeping the calcium hot, fresh air is allowed to enter and the pumping is continued. This operation should be repeated several times to sweep the water vapor from the walls of the container, after which the stop-cock is permanently closed.

With reference to the efficiency of the device it may be stated that, after the gases have been absorbed by the calcium, the vacuum is so high that the discharge from an induction coil passes through the 5 cm air space between the electrodes *p*, *p*, Fig. 1, in preference to passing through the evacuated tube.

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[8] B. S. Bulletin 11, p. 156, 1914; 13, p. 446, 1916 ("Mounting for Vacuum Thermopiles").

As mentioned elsewhere, an electric heater may be used, but it requires close attention to avoid overheating of the quartz tube. It is not absolutely necessary to use a pump to start the evacuation. However, explosions may result from the expulsion of the absorbed water vapor, acetylene, etc., and the resultant increase in gas pressure, when the calcium is first heated in a closed tube at atmospheric pressure. Hence provision must be made for increase in gas pressure during the initial heating of the calcium in a closed tube containing air at atmospheric pressure. It is to be remembered that the chemical action begins at about 300° C and is most effective at a low red heat.

The first samples of the vacuum chamber, with attached calcium evacuator, described in the foregoing pages, were prepared about 7 years ago and taken 3,200 miles to an observatory without mishap. This experience showed that equipped with several vacuum radiometers of this type one can go to the remotest station without taking a pump.

A further improvement will come from the discovery of a cement for securing the windows which will not give off vapors, and thus increase the efficiency of the device.

IV. CONCERNING RADIATION SENSITIVITY TESTS OF VACUUM RADIOMETERS

It is beyond the scope of this paper to describe fully the methods of calibration of radiometers and the various methods which may be employed to test the variation in their radiation sensitivity.

In this connection it is to be remembered that the sensitivity of the auxiliary Thomson galvanometer varies with the diurnal change in terrestrial magnetism, etc. The current sensitivity may therefore be tested by a simple device described in previous papers.[9]

The radiation sensitivity of a bolometer and to a less extent a thermopile (because of its greater heat capacity) is a function of the kind and the pressure of the residual gas. The sensitivity of a

[9] B. S. Bulletin 10, p. 19, 1913; 11, p. 171, 1914. (Specification of the Radiation Sensitivity of a Thermopile).

bolometer is 2.5 as great in hydrogen as in air at the same gas pressure. It has been found by Reinkober[10] and also by the writer that the radiation sensitivity of the Ruben's thermopile (of wire 0.15 mm in diameter) in a vacuum is only about 1.5 times that in air. The radiation sensitivity of the present thermopile is doubled in a vacuum. In this instrument the sensitivity varies but little during the course of a day's work. In making transmissivity (and reflectivity) measurements it is sufficient to have the radiation sensitivity remain constant while making the two sets of measurements, viz., (1) the intensity of the radiation transmitted through the substance, and (2) the intensity of the direct radiation.

If it is desired, the variation in radiation sensitivity of the vacuum thermopile (and of vacuum radiometers in general) may be determined by means of a standard incandescent lamp,[11] or other suitable means.

In conclusion it is relevant to emphasize the fact that the increase (say 50 to 100 per cent) in radiation sensitivity by placing the ordinary laboratory radiometer (thermopile) in a vacuum is relatively small and that the chief gain lies in elimination of unsteadiness of the galvanometer reading by elimination of convection currents. This is easily accomplished by keeping the air pressure less than 0.1 mm.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
Mar. 4, 1921.

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[10] Reinkober, Ann. der Phys., (4) 34, p. 343; 1911. Lebedew (Ann. der Phys., (4) 9, p. 209; 1902) found that above 5 mm (also below 0.01 mm) a variation in gas pressure has but little effect upon the sensitivity.

[11] McCauley, Astrophys. J., 37, p. 164; 1913.

THE ORIENTATION OF THE GRAINS IN A DRIED PHOTOGRAPHIC EMULSION*

BY
L. SILBERSTEIN

In my first paper on this subject the number of grains (small plates) whose inclination in the initial state falls within the limits θ and $\theta+d\theta$ was, through an obvious inadvertency, assumed to be $\frac{2}{\pi} N d\theta$, as if the plate-normals were confined to a plane. The directions of these normals being haphazardly distributed in *space*, the number in question is manifestly equal N times the ratio of the area of the spherical zone $d\theta$ to that of the hemisphere, *i.e.*,

$$N \sin\theta \, d\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (a)$$

This then has to replace the first (and only) expression on p. 171.

The remainder of p. 171, as well as the whole of p. 172 and p. 173 up to section 1 are unaffected, while the formulae concerning the orientation in the final, dry state of the emulsion are to be modified as follows:—

1. By the "sine-law," $\sin\theta' = k \sin\theta$, formula (1), we have, as before, $\cos\theta' = \sqrt{1-k^2\sin^2\theta}$, and therefore, by (a), for the efficiency coefficient κ, as defined on p. 171,

$$\kappa = \int_0^{\pi/2} \sin\theta \sqrt{1-k^2\sin^2\theta}\, d\theta$$

or, putting $\cos\theta = u$,

$$\kappa = \int_0^1 \sqrt{1-k^2+k^2u^2}\, du.$$

The integral is easily evaluated and gives the efficiency coefficient

$$\kappa = \tfrac{1}{2} + \frac{1-k^2}{4k} \log\frac{1+k}{1-k} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (I)$$

or, developing the log,

$$\kappa = 1 - \frac{k^2}{1.3} - \frac{k^4}{3.5} - \frac{k^6}{5.7} - \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (I')$$

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* Corrigenda to a paper of equal title published in the March number of this Journal, pp. 171–177, 1921.

of which, the contraction ratio k being a small fraction, the first two terms will suffice in practice. For $k=0$ we have $\kappa=1$, an obvious result, and for $k=1$ (initial state) $\kappa=\frac{1}{2}$, as it should be. The whole range of κ is thus contained between $\frac{1}{2}$ and 1.

If the thickness ϵa of the grain-plates is taken into account (p. 174), the efficient area is increased by $a\epsilon k$. $\sin^2\theta=\frac{1}{2}a\epsilon k$, and the correction term $\frac{1}{2}\epsilon k$ is to be added to the value of κ given above.

2. By the alternative assumption on the contraction process, the "tangent-law," $\tan\theta'=k\tan\theta$, we have $\cos^2\theta'=1:(1+k^2\tan^2\theta)$, as before, and, by (a), substituting $u=\sin^2\theta$,

$$\kappa=\int_0^{\pi/2}\frac{\sin\theta\cos\theta\,d\theta}{\sqrt{1-(1-k^2)\sin^2\theta}}=\tfrac{1}{2}\int_0^1\frac{du}{\sqrt{1-(1-k^2)u}}.$$

Thus the efficiency coefficient corresponding to the tangent-law is simply

$$\kappa=\frac{1}{1+k}\qquad\text{(II)}$$

This gives for $k=1$ and $k=0$ the values $\kappa=\frac{1}{2}$ and 1, as above, while for intermediate k the values of κ are smaller than those given by (I). For contractions such as $k=\frac{1}{10}$ the two κ-values differ only by about 9 percent. The following table (replacing that of p. 175) gives κ on either assumption for a few values of k:

k	=1.0	0.5	0.3	0.2	0.1	0.0
κ(I)	=0.5000	0.9125	0.9695	0.9866	0.9967	1.0000
κ(II)	=0.5000	0.6667	0.7692	0.8333	0.9091	1.0000

(The remainder of p. 175 is unaffected.) On the sine-law the total range of inclinations θ' in the dry film is 0° to $\alpha=\arcsin k$, and on the tangent-law, 0° to 90° (as before).

Finally, concerning the distribution through these angle ranges (p. 176), if $N'd\theta'$ be the number of grains whose inclinations in the final state fall within the interval θ' to $\theta'+d\theta'$, we have

$$N'd\theta'=N\sin\theta\,d\theta.$$

Thus $\left(\dfrac{d\theta}{d\theta}\text{, being as on p. 176}\right)$ we have, on the *sine-law*, the final distribution law

$$N'd\theta'=N\frac{\cos\theta'\sin\theta'\,d\theta'}{k\sqrt{k^2-\sin^2\theta'}},\qquad 0<\theta'<\arcsin k\qquad\text{(4)}$$

and, on the *tangent-law*,

$$N'd\theta' = N\,\frac{\sin\theta' d\theta'}{k^2\cos^2\theta' + \sin^2\theta'}, \qquad 0<\theta'<\frac{\pi}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4^t)$$

Formula (4^s) gives, for the number $n'(\theta')$ of grains whose inclinations fall within the interval 0 to θ',

$$n'(\theta') = \int_0^{\theta'} N'd\theta' = \frac{N}{2k}\int_0^{u}\frac{du}{\sqrt{k^2-u}},$$

where $u = \sin^2\theta'$, *i.e.*,

$$n'(\theta') = N\left[1-\sqrt{1-\frac{\sin^2\theta'}{k^2}}\right] = N\,[1-\cos\theta], \quad 0<\theta'<\alpha \quad \ldots\ldots\ldots\ldots (5^s)$$

as it should be, for all these grains belonged originally to the range 0 to $\theta = \arcsin\left(\frac{\sin\theta'}{k}\right)$. Similarly, in the case of the tangent-law,

$$n'(\theta') = N\,[1-\cos\theta], \qquad \theta = \arctan\left(\frac{\tan\theta'}{k}\right), \quad \ldots\ldots\ldots\ldots\ldots\ldots (5^t)$$

valid for any θ' between 0 and $\frac{\pi}{2}$.

By (5^s) we have, for example, for $\sin\theta' = \frac{1}{2}k$,

$$n' = N\left(1-\frac{\sqrt{3}}{2}\right),$$

i.e., 13.4 per cent of all grains (the remaining 86.6 per cent being contained between $\arcsin\frac{k}{2}$ and $\arcsin k$). For small k formula (5^s) can be written, approximately,

$$n'(\theta') = \tfrac{1}{2}N\left(\frac{\theta'}{k}\right)^2.$$

The tangent-law formula (5^t) gives for $\theta' = 45°$, for instance, when $\tan\theta = 1/k$,

$$n'\left(\frac{\pi}{4}\right) = N\left[1-\frac{k}{\sqrt{1+k^2}}\right].$$

Thus, if $k = \frac{1}{10}$,

$$n'(45°) = 0.905N,$$

that is to say, 90½ per cent of the grains have inclinations contained between 0° and 45°, and the remaining 9½ per cent between 45° and 90°.

EASTMAN KODAK CO
ROCHESTER, N. Y

SOME MAJOR PROBLEMS IN PHOTOMETRY*

BY
E. C. CRITTENDEN and J. F. SKOGLAND

I. INTRODUCTION

The purpose of this paper is to present a general statement of certain important problems in practical photometry. The questions raised are not new, but have been under discussion for many years. Notable progress has been made, but some difficult problems still remain. For workers in optics outside of this special field it appears worth while to give a brief statement of the present situation as a reminder that these problems still exist and that investigators in related fields may be able to make material contributions to their solution.

In some respects photometry is now on a very satisfactory basis, particularly in its industrial applications, so that it may perhaps be considered fairly advanced as an art, if not so far along as a science. The value of the fundamental *unit*, the candle, is firmly established and undoubtedly can be maintained for a long time with a high degree of certainty. A group of about fifty carbon-filament incandescent electric lamps now maintains this unit at the Bureau of Standards. These lamps are carefully preserved and are burned only at such times as it is considered necessary to recalibrate the secondary standards which are used for routine tests. Consequently there is little danger of any appreciable drift in their mean value, and it is believed that the unit can be maintained for a very long time to within one-tenth per cent for the average of all, and to within two-tenths per cent for individual groups of from ten to twelve lamps. By comparison with such standards the more common sources can be measured with an accuracy which is fairly satisfactory.

From another point of view, however, more in line with scientific considerations, the whole field of light measurement is in an

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[1] Presented at the December, 1920, meeting of the Optical Society of America. Published with the permission of the Director of the Bureau of Standards.

unsatisfactory state. In the first place there is no really reproducible fundamental standard. Even if it be granted, that such a standard is not vital to successful photometric measurements, and that an arbitrary unit well maintained meets the requirements of practice, there remains a second difficulty in that no general agreement has been reached regarding the methods which should be used in measuring lights which differ in color from that given by the standard source.

There are therefore two problems presented, which in their broader sense embrace all photometry. The first is the establishment of a reliable and reproducible primary standard of light; the second, the specification of the methods to be used in comparing other lights with that of the standard. For the present purpose it will suffice to outline the actual status of these problems without attempting to indicate what may be their ultimate solutions.

II. PRIMARY STANDARDS

As a preliminary to the discussion of standards it may be well to emphasize the distinction between the *unit* of measurement and the *standard*. Some years ago all the important nations except the Germanic group came to an agreement on a common unit, and the relation between this unit and the German Hefnerkerze was established as the simple ratio of 10 to 9. Incidentally in all countries the name given to the unit is *candle*, or an equivalent word, but no country has actually retained the candle as a standard. In passing from the candle as a standard, however, each country has followed an independent course, and consequently the nominal basis of the common unit is different in each of the four great nations, France, Great Britain, Germany, and the United States. The first three of these adopted different flame lamps as standards. If the units were actually redetermined from these standards which are nominally accepted as fundamental, it is therefore probable that differences would appear among the units thus derived.

In France the classic basis for the value of the candle is Violle's platinum standard, but no one ever succeeded in reproducing

Violle's results, and it is now recognized that this standard is far from being exactly reproducible. In practice Violle's determination of the unit was applied by using the Carcel lamp with the value which he assigned to it, and until recently this lamp has been accepted as at least an intermediate reference standard. The law of April 2, 1919, while referring to the Violle standard, specifically says that the unit, the Bougie Décimale, is represented practically and in a permanent manner by means of incandescent (electric) lamps deposited in the Conservatoire Nationale des Arts et Métiers. In Great Britain the candle was superseded by the Harcourt 10-candle pentane lamp, but since it is at least difficult to reproduce such lamps exactly, the standard most generally accepted is not the pentane lamp in general, but a particular lamp of this kind at the National Physical Laboratory. Moreover, the standards actually used at the National Physical Laboratory are electric incandescent lamps. Values were originally assigned to these lamps by comparison with the pentane lamp mentioned, but published records indicate that only two series of such comparisons have been made, so that in effect the unit is maintained by the electric standards. In Germany the Hefner lamp is presumably still the generally recognized standard. Its extreme simplicity gives it an exceptional degree of reproducibility, but with so simple a lamp one can obtain only a very yellow light of low intensity, and in transferring values to standards of considerably different color and intensity such as are necessary for practical use grave difficulties are introduced. These difficulties have become progressively greater as efficiencies of ordinary illuminants have been improved, with a consequent greater departure from the color of the flame lamps, particularly the Hefner.

In addition to the uncertainties inherent in these various flame standards, all of them show systematic variations resulting from atmospheric conditions. Consequently in every case where definite values have been established, the actual light output of the flame standards under different conditions has been determined by means of incandescent electric lamps, which are not affected by these conditions. The electric lamps have proved so much more reliable and convenient than the flames that there has been

a marked tendency to use them as the real standards, comparisons with the flames being rarely made and then usually only as approximate checks. In fact the international agreement mentioned has been obtained and is maintained by means of comparisons between groups of electric standards.

This procedure is in part analogous to that followed in the case of the international ohm, which is maintained by means of wire resistance standards whose value is derived from mercury ohms set up at infrequent intervals, but there is the very important difference that there exists no reproducible primary standard of light analogous to the mercury ohm. In the adoption of the unit of candlepower in the United States this difficulty was faced, and it was decided not to adopt any of the existing nominal primary standards, but to base all values on a large group of electric incandescent standards pending agreement on a more satisfactory primary standard.

Some fourteen years have passed since this decision was made. During that time extensive experimental studies of the various flame standards have been made, and the net result of these investigations has been a confirmation of the wisdom of the course taken. It appears that the flame lamps at best can serve only as a comparatively rough check to show any considerable drift of the electric standards. Tradition and national pride make it difficult for any country to give up its old standards, but so far as the United States is concerned the field is entirely open, and internationally the time appears favorable for the consideration of new proposals in this field.

Without attempting to review the many proposals and the relatively few direct experimental contributions made to this subject during the past decade, it may be said that progress in related fields of work has undoubtedly brought nearer the possibility of two solutions of the problem, either of which would be satisfactory. One of these is the precise specification of the conditions under which some radiator, such as a black body, will give light of a definite intensity; the other is the accurate measurement of radiant power of specified quality incident on a surface. The first appears the more promising, but it yet remains to be shown

experimentally that the conditions of operation of the radiator can be controlled so as to reproduce light intensities with a certainty of one per cent or better. At the present time any procedure which would promise this degree of certainty would receive serious consideration, but as indicated above, in order to replace the present electric lamps as the ultimate basis of reference, a standard must be capable of reproducing values within a few tenths of one per cent over a long period of time.

III. GENERAL PHOTOMETRIC METHODS

1. *Significance of "Brightness."* When a unit of light has been established and standards obtained which maintain or reproduce this unit with the required accuracy, questions still arise regarding the meaning of "measurements" of light, and the significance of the results obtained by different methods.

So long as the lights dealt with are of about the same color it is easy to determine quantitative relations between them, and it is generally agreed that measurements shall be made by methods which consist essentially of obtaining an equality of brightness on two similar surfaces. But when we have to deal with lights differing considerably in color it becomes doubtful what is meant by equal "brightness." Although it is convenient to describe lights (or colors) by reference to three properties which may be called hue, saturation and brightness, this analysis is more or less arbitrary, since sensations are not really separable into components. In assigning values to such an assumed property of a sensation we are at best on rather uncertain ground. Moreover it is certain that no one numerical value can represent the relative usefulness or effectiveness of two lights which differ in color, since their relative values must depend on the purposes to be served and on the conditions under which they are to be used. But for the very purpose of studying relative usefulness or effectiveness of different lights under different conditions it is necessary to have some means of describing them quantitatively. If radiometric measurements were not so difficult to make there would be decided advantages in comparing lights on the basis of radiant power, supplemented by suitable qualitative specifications. In practice,

however, the most convenient and useful way to compare lights quantitatively is by a measurement of the somewhat indefinite property called "brightness." The unsettled question is, by what method this comparison is to be made.

2. *Ordinary Photometric Measurements.* As a natural development the same types of photometers used for measurements involving no difference of color have been commonly used also for measurements in which color differences occur. If this procedure gave satisfactory and consistent results it would have been accepted without question, but such serious difficulties rise when the color differences are considerable that the results obtained are inconclusive and unsatisfactory, and it appears that if any systematic and general method of photometry is to be established it must be something other than the method of direct comparison by the usual photometer.

It should be recognized that the ratio of the brightness of two lights which differ in color is not a definite and single-valued quantity, but depends upon the conditions of observation as well as upon the characteristics of the individual observer's eye. In particular, in order to make the results definite it is necessary first to specify more or less precisely the size and brightness of the photometric field, which determine the parts of the retina used and its level of adaptation. This condition is easily fulfilled. It is also necessary to include observations by a sufficient number of individuals to approximate the results which would be obtained by the average, or normal, observer. This last condition is difficult to realize in practice in any one laboratory when the ordinary photometer is used. In the first place consistent results can be obtained only by selected observers who have had considerable practice in the particular kind of measurements to be made, and no laboratory can be expected to have many observers who have sufficient experience to make satisfactory measurements. There is also a natural tendency in selecting and training observers to bring the new ones into agreement with the more experienced, and thus to perpetuate any peculiarity shown by the original staff. Experience has shown that groups of observations which are apparently very reliable because of their high precision may be

decidedly inaccurate when referred to values established by larger numbers of observers. This difficulty can be obviated to some extent by the collaboration of a number of laboratories; and for some kinds of measurements, such as the comparison of different types of incandescent electric lamps, fairly satisfactory results can be obtained by establishing secondary reference standards of the different types. This course has already been followed to some extent in international comparisons through which approximate agreement on certain tungsten filament standards has been obtained, thus supplementing the original agreement on a basic unit preserved by means of carbon filament lamps. The values for these secondary standards are originally determined by comparison with the fundamental standards, but the time and labor required to obtain a satisfactory agreement are so great that values once determined and agreed upon are likely to be considered as practically independent. Such multiplication of standards, while serving to meet the more urgent commercial needs, is limited in its application. There is always doubt as to the certainty with which the values can be reëstablished by reference to the fundamental standards, even though the color differences bridged are the relatively small ones represented by incandescent lamps of different efficiencies.

3. *Flicker Photometry.* As a more general solution it has been proposed to use the flicker photometer, which presents the two photometric surfaces alternately in succession in the same place instead of simultaneously side by side. Its advantages depend upon the fact that the surfaces can be interchanged at such speeds that the disturbing hue differences are no longer seen while differences of brightness still show in the form of a perceptible flickering of the light in the field. Thus a setting for equal brightness may be made by finding the point at which the flicker disappears. Whether the "brightness" thus determined is exactly the same thing as that found by the ordinary photometer is a disputed question, but since brightness must be rather arbitrarily defined in either case this particular question does not appear to be of much importance. At any rate, a comparison of values obtained by these two methods indicates that approximately the same quan-

tities have been evaluated, and the differences are not so great as those which appear, for example, between acuity and equal-brightness results.

A choice of conditions giving exact agreement between the two methods would, of course, be most desirable. Ives, who has done most to develop the possibilities of the flicker method, decided from his experiments that if a small field (2°) and a fairly high brightness (2.5 millilamberts) were used on each photometer the same results would be obtained on the two. Such independent evidence as is available seems to corroborate this conclusion, but the conditions specified are considerably different from those of the ordinary standard photometer, and results obtained with it differ accordingly. If the new conditions proposed are adopted, some values already established under the older conditions will presumably have to be modified slightly, but the changes required are of the same order of magnitude as the uncertainty which would be involved in repeating the old determinations.

The flicker instrument is more complicated and consequently more difficult to keep in good working order. Measurements with it are also somewhat slower and considerably more fatiguing. It is essentially an instrument for the research or standardizing laboratory, rather than for general use. Its great advantages are that definite and fairly precise measurements can be made on lights of any color, and that erratic variations of individual judgment are reduced, so that from an observer's results in one kind of measurement his performance in another kind can be accurately predicted. This makes it possible to select observers systematically to represent proper average characteristics. Such averages or normals for any desired color can easily be established by large numbers of observers, thus avoiding the assumption that the work of a few specially selected observers is a fair representation of average visual characteristics. Although full experimental data are lacking, experience so far available indicates that, with the flicker photometer properly used, different laboratories may readily obtain good agreement in measurements involving color differences of any degree.

4. *Use of Visibility Curves.* In a sense the ultimate result of any method of comparing the intensity of lights of different colors is expressed in a "visibility curve," that is, a curve showing the relative visual effect of radiant power of different wave lengths throughout the spectrum. Given such a curve and the spectral transmission curve of any absorbing screen, the determination of the total transmission becomes merely a matter of calculation or of graphic determination.

Visibility curves differ considerably for different conditions of measurement, as well as for different individuals under given conditions. Curves obtained by different observers under the same conditions scatter over a range much greater than the range of the average curves obtained by different methods. Moreover, the experimental difficulties in determining these curves are great, and systematic errors of considerable magnitude may occur in the best determinations. Priest has therefore proposed as a solution of the whole problem of heterochromatic photometry the adoption of an arbitrary normal visibility curve to be used especially in calculating the transmission of quartz-nicol combinations which can be chosen to equalize almost any color difference, thus reducing all photometric observations to a color match.

This procedure is undoubtedly practicable, and such use of visibility curves already gives results in some cases more reliable than those obtained by direct measurements of the usual kind, but it is hardly to be expected that direct measurements in general will be discarded or given a secondary position. Instead of this, a visibility curve which may be adopted as normal will presumably be adjusted to obtain results in concordance with direct measurements, and this adjusted curve will be used within the limits of its applicability. For example, such a curve when established will also constitute the transmission curve of solutions for use with non-selective radiometers as physical photometers. The basis of photometry must, however, continue to be evaluation by the sense of sight, and this is the fundamental argument for the adoption of a visual method as the basic one.

5. *Choice of Basic Method.* At present there appear to be three possible methods of meeting the difficulties of heterochromatic

photometry. First, the usual standard photometer may be retained as the basis of all measurements, the effects of its unsatisfactory features being in part avoided by the establishment of secondary standards for lights of various colors and by calculations from a visibility curve in extreme cases. Second, the flicker photometer may be adopted as the standard method for all color-difference measurements, visibility curve calculations being used as auxiliary only. Third, by some adjustment of conditions all three methods may be brought into approximate agreement.

The last is of course the solution to be desired, but it is not clear how it can best be brought about. It is certain that for some time to come the question of the use of the flicker photometer will be a live one, and every photometric laboratory may well acquire some acquaintance with this instrument, which in our experience has gained greater confidence in regard to certainty of results and adaptability to all measurements as its use has become more common in the laboratory.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A REMEASUREMENT OF THE RADIATION CONSTANT, h, BY MEANS OF X-RAYS[1]

BY

WILLIAM DUANE, H. H. PALMER and CHI-SUN YEH

Since the discovery of the fact that the short wave-length limit of the continuous x-ray spectrum obeys the quantum law,[2] $Ve = h\nu$, this phenomenon has been used by a number of experimenters[3] to determine the value of h. The most accurate measurement of h in this laboratory was made by Blake and Duane,[4] who obtained the value $h = 6.555 \times 10^{-27}$, with $e = 4.774 \times 10^{-10}$, (Millikan's value).

Some years ago our laboratory purchased 200 high resistance manganin wire coils for the purpose of measuring the voltages applied to x-ray tubes by comparing them directly with the electromotive force of a standard cell by the simple potentiometer principle. We hoped by this means to measure the voltages with such accuracy that we could neglect the errors in them in comparison with those in the other quantities entering into the quantum equation. It now appears that we can do this, and we are able, therefore, to test the quantum law under various experimental conditions and measure the value of h with greater precision and accuracy than before.

In addition to the voltage measurements improvements have been made in the measurements of the frequency, ν. We have used a new x-ray spectrometer with scales that appear to be somewhat better than the old ones (there being no appreciable eccentricity) and with a new calcite crystal. Further, the x-ray tube had a long side arm attached to it with a very thin mica window W at its end. This extended out toward the spectrometer (Fig. 1). The

[1] A paper presented to the American Physical Society at its Washington meeting, April, 1921.

[2] Duane and Hunt, Phys. Rev., Aug., 1915, p. 166.

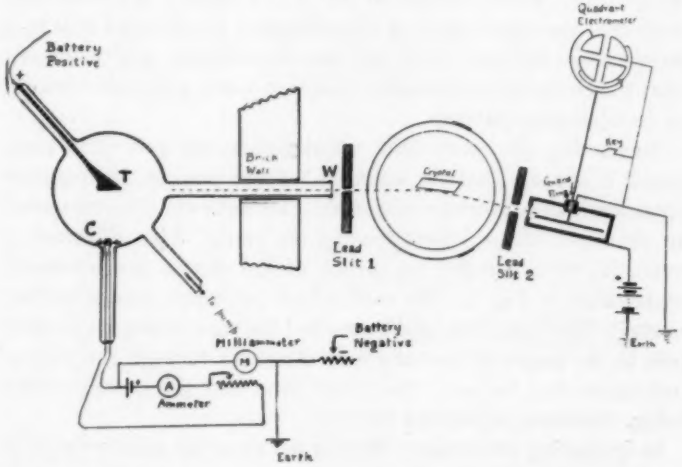
[3] For references etc., see reports in the Jahrbuch der Radioaktivität for Dec., 1919 by E. Wagner and for Dec. 1920 by R. Ladenburg and also a report on Data Relating to X-Ray Spectra by William Duane, published by the National Research Council.

[4] Phys. Rev. Dec. 1917, p. 624.

fact that the mica allowed a much larger fraction of the x-radiation to pass through enabled us to use a narrower slit (Slit 1), which reduced the correction that must be applied for the slit's width.

In estimating the difference of potential, V, through which the electrons in the x-ray tube pass we must take account of the following considerations. The x-ray tube was constructed in our laboratory, and was permanently connected to a series of three pumps—a mercury vapor diffusion pump, a Gaede rotary mercury pump and a rotary oil pump. We had no difficulty in exhausting the tube so that no appreciable current could be ascribed to the

FIG. 1



ionization of the residual gas in it. The x-ray tube contained a Coolidge cathode, which consists of a spiral tungsten wire that can be heated by means of a current coming from a small auxiliary storage battery. The electrons emitted by this wire, when hot, carry the current through the tube between the anode, T, and the cathode, C, (Fig. 1). Owing to the heating current flowing through the spiral wire the points on it are not all at exactly the same potential. An examination of the electrical connections represented in Fig. 1 shows that the points on the spiral wire

had potentials above that of the gas pipe (marked earth in the figure), to which the electrical circuits were connected by a soldered junction. Since in the quantum relation we deal with the maximum frequency of the x-rays in the continuous spectrum, we must estimate the maximum difference of potential between the anode and any point on the cathode. This point is the end of the spiral nearest to the wire connecting it to earth. Measurements by means of a high resistance voltmeter indicated that this point had a potential about 0.7 volt above that of the earth under the conditions during the experiments. We have neglected this difference of potential in comparison with that applied to the x-ray tube, which amounted to 24,413 volts. We have also neglected the effect of any Volta difference of potential that may have existed between the anode and the cathode, and of the fact that the electrons leave the hot tungsten spiral with velocities due to its high temperature.

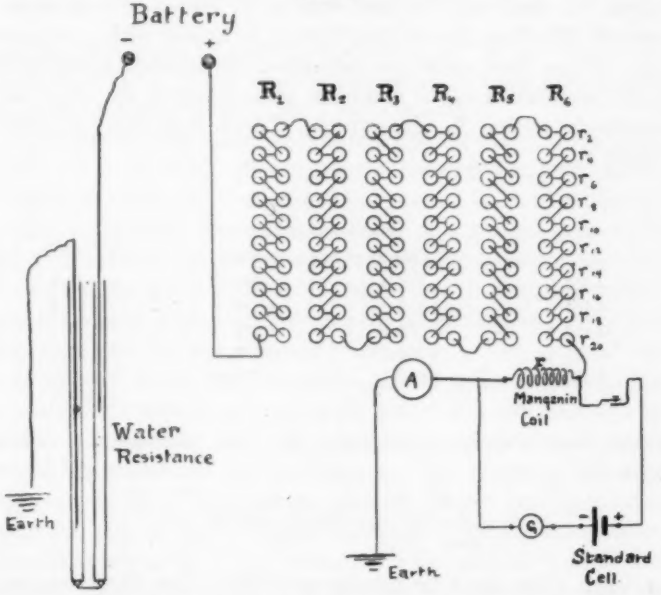
Neglecting the corrections mentioned in the preceding paragraph it appears that the value of V to be used in the quantum equation is the difference of potential between the wire connected to the anode of the x-ray tube and the earth. This difference of potential we measured by means of the simple potentiometer represented in Fig. 2. We used a high potential storage battery to excite the x-ray tube, and connected the wire joining its positive pole to the anode of the tube with the earth through the various resistances R_1, R_2, etc., the x-ray tube and the potentiometer being, therefore, in parallel circuits.

In insulating the various circuits between the positive pole of the battery and the earth we used the same precautions as are employed in making ionization measurements with an electrometer or an electroscope. All instruments, ammeter, galvanometer, standard cell, etc. rested on hard rubber supports the surfaces of which we cleaned carefully by means of sandpaper. No wires touched the tables or walls of the room etc. They were supported on silk threads. The various resistance coils, r, r_1, r_2, etc. hung from shellacked glass rods, which in turn rested in six wooden frames, forming six sections, R_1, R_2, etc., of the main resistance. These six wooden frames hung by silk threads from a

broad, horizontal board, which served not only as a support, but also to keep the dust off of the insulating surfaces. We tested carefully the insulation. In particular, on closing the circuit to the high potential battery no permanent deflection of the galvanometer, G, occurred, if the key in its circuit was open. A transient deflection occurred, doubtless due to electrical induction.

In order to compare the difference of potential applied to the x-ray tube with the emf of the standard cell we do not have to

FIG. 2



know the absolute values of the resistances. Their ratios suffice. To determine these ratios we proceeded as follows. We measured the resistance of r_{20} by means of a standard Wheatstone bridge, and found it to be $r_{20} = 50{,}214.6$ ohms. We do not have to examine the accuracy of this value. We assume it to be correct, and compare the resistances of the other coils with it. To do this we formed a Wheatstone bridge with three of the coils of section R_5,

and substituted in it, as the fourth arm, each one of the coils of section R_6 in succession. The galvanometer had a sensitiveness of 10^{-11} ampere per mm of scale deflection, and we used as much of the high potential battery as would send a current of 4.0 milliamperes through each arm of the bridge. (About 4 milliamperes passed through the potentiometer circuit during the experiments with x-rays). With each of the coils of section R_6 in the bridge we determined the resistance that had to be added to it in order to balance the bridge. These resistances could be measured to within about 0.1 ohm, and, as they were to be added to resistances of over 50,000 ohms they did not have to be known with great accuracy. From these values we calculated the numbers of ohms that must be added to r_{20} to make the sums equal to the other resistances in section R_6 respectively. These resistances appear in Table 1.

TABLE 1

Resistances of Coils

Values of $r_x - r_{20}$ in ohms

+304.9	+271.4	+203.8	+321.4
+101.6	+174.6	+175.4	+ 82.3
+133.4	− 35.8	+264.5	+ 39.1
− 51.1	+161.7	+330.8	+ 24.2
+151.8	−220.7	+122.4	
			+2555.8

From these values we compute the total resistance of section R_6 to be

$$R_6 = \sum_{x=1}^{x=20} r_x = 20 \times r_{20} + 2555.8 = 1{,}006{,}848,$$

and this value must be correct to within a few ohms, assuming always that r_{20} has the above mentioned value.

To measure the resistances of the other sections R_1, R_2, etc. we formed Wheatstone bridges with the several sections as arms, and by substituting R_6 and each of the other sections in them we determined the resistance that must be added to R_6 to equal the resistance of each of the other sections. Table 2 contains these values.

TABLE 2

Resistances of Sections

Values of $R_y - R_6$ in ohms

−866.4	+899.8	+452.3
−378.6	−1691.	
		−1584.

They could be measured to within about one ohm, so that the total resistance of all six sections amounts to

$$R = \sum_{y=1}^{y=6} R_y = 6R_6 - 1584 = 6{,}039{,}504 \text{ ohms},$$

and this appears to be correct to within a few 10's of ohms. In these experiments, also, the current through each arm of the bridge was maintained at about 4 milliamperes.

It remains to determine the resistance of the coil r, to the ends of which are attached the terminals of the standard cell circuit. The ratio of r to r_{20} we measured by means of a Wheatstone bridge in which the other two arms consisted of standard coils of 50 and 10,000 ohms respectively. The values of these coils were certified by the Bureau of Standards, and further their ratio was carefully checked in Professor H. N. Davis' laboratory by comparison with a number of other certified standards. The value of the ratio we used is 199.998. Except r, all the coils we used in our measurements were several years old and it does not seem unreasonable to suppose that they remained sufficiently constant during the course of our experiments. We wound r, however, expressly for these experiments, and we did not feel justified in assuming that it would remain constant. We therefore determined for each of our measurements of h the number of ohms that had to be added to r_{20} in order to make its ratio to r equal the ratio of the standard coils as given above. These resistances, called x, appear in the third column of Table 3. The fourth column contains the values of r calculated from x, assuming that $r_{20} = 50{,}214.6$ ohms, as above. In these experiments we found that, if the coils r and r_{20} had been heated by a current of 4 milliamperes for some time, the value of x was 1 ohm larger than when cold. We recorded the larger value.

TABLE 3

Resistances and Differences of Potential

Date	Temperature	Resistance x	Resistance r	Emf. of Standard Cell	Difference of Potential, V
March 15	21°	185.1	252.014	1.01863	24412.4±2.0
March 21	22°	185.3	252.015	1.01863	24412.3
March 30	20°	183.8	252.007	1.01864	24413.3
April 5	19°	183.8	252.007	1.01864	24413.3

By taking the ratio of (R+r) to r we get the ratio of the voltage applied to the x-ray tube to the emf of the standard cell under the condition that no current passes through the galvanometer circuit. During the course of an experiment one observer watched the galvanometer and kept changing the variable water resistance (Fig. 2) so as to keep the galvanometer reading as near zero as possible. Deflections of several mm could not be avoided, but, as the galvanometer had a sensitiveness of 10^{-11} ampere and the resistance in its circuit was not very large, these variations did not correspond to appreciable fluctuations of the voltage.

With the above described measurements and procedure it seems reasonable to assume that we knew the value of the voltage and maintained it constant during an experiment to within an accuracy represented by the accuracy with which we know the emf of a standard cell. We had two Weston unsaturated cells at our disposal each with a certificate from the Bureau of Standards, in which the accuracy was stated to be one part in ten thousand. We compared these two cells with each other during each experiment by means of a potentiometer. In each case the difference between them agreed with the difference between their certified values to within the accuracy claimed in the certificates. In estimating the emf of the cell actually used in a measurement we gave equal weights to the certificates. The mean value of this emf appears in column 5 of Table 3. The sixth column contains the differences of potential, V, applied to the x-ray tube. We think these values are correct to within about two volts.

In determining the position of the short wave-length limit of the continuous spectrum (the value of ν to be substituted in the quan-

tum equation) we proceeded as in previous experiments except that we used a slightly different method of estimating the slit correction. While one observer watched the galvanometer, G, and kept the voltage constant another measured the rate of deflection of the quadrant electrometer, which determines the current in the ionization chamber (Fig. 1). Curves A and B in Fig. 3 represent these ionization currents (measured on both sides of the zero of the instrument) as functions of the readings of the verniers on the scale that fixed the positions of the reflecting crystal. The horizontal portions of these curves correspond to the natural leak of the instrument and to that due to stray radiation etc. The

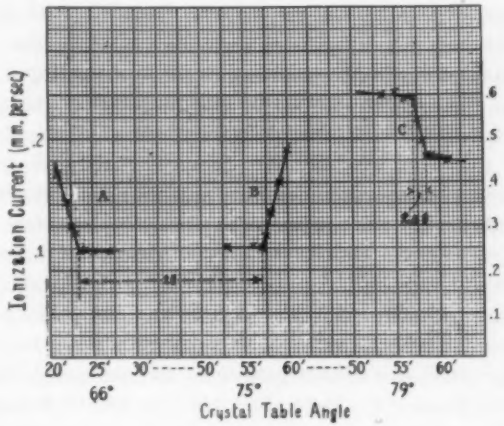


FIG. 3

inclined portions correspond to x-radiation reflected from the crystal. The inclined portions recorded in the figure occupy angular breadths of 2′ and 3′. Since the glancing angle, θ, amounts to about 290′, this means that the part of the radiation actually measured in this experiment lay within about one per cent of the end of the continuous spectrum. The limit of the spectrum appears, therefore, very sharply defined. We believe that we can estimate the minimum value of θ in a good experiment to within about 7″ of arc.

Owing to the fact that the rays passing through the slit 1 are not all exactly parallel to each other a small correction has to be added to the angle as measured from curve A to curve B. The correction to be added to the double glancing angle, 2θ, equals the greatest angle between any of the rays in the x-ray beam. Slit 2 being wide enough to allow the whole reflected radiation to enter the ionization chamber, the breadth of the source and the width of the slit 1 determine this angle. Its magnitude equals the angular breadth of the slit 1 as seen from the target plus the angular breadth of the focal spot on the target as seen from slit 1.

In our experiments the angular breadth of the focal spot as seen from the slit 1 was small. We determined its magnitude as follows. Before the tube was constructed the surface of the target was ground very smooth. After the tube's construction, but before it had been put in place, we estimated by reflecting a beam of light from the smooth surface of the target that the plane of this surface would pass through a point about 5 mm from the centre of the mica window, W. Since the distance from the target to the window was about 720 mm, this means that the rays that passed through the centre of the slit 1 left the target making an angle of about 24′ with its surface. From the slight marks on the target we estimated that the breadth of the focal spot was 8 mm. Since from slit 1 this is seen at an angle of 24′ its apparent angular breadth is about 15″.

We determined the width of slit 1 by the method described in the Physical Review for December, 1917, p. 630. In one experiment we found that we had to decrease the width of the slit .313 mm before the current in the ionization chamber ceased, and this is the effective width of the slit. Dividing it by the distance from the slit to the target, 760 mm, we find that the angular width of the slit as seen from the target is 1′ 25″. Adding the angular width of the focal spot we get 1′ 40″ as the correction, $2\Delta\theta$, to be added to the double glancing angle, 2θ, in this particular case.

The geometry of the tube and spectrometer determines the correction as calculated above. It might well be that other causes, such as irregularities in the crystal planes, would introduce errors. It becomes, therefore, desirable to have another method

of estimating the correction,—if possible, one that would give us an upper limit for its value, since the geometry determines a lower limit. We have examined some of the emission lines in the L series of tungsten to see if their widths corresponded with the geometry of the slit and source. In every case the peak on the ionization curve that represented an emission line appeared to be somewhat broader than would be expected from the widths of the slit and source. On putting ethyl-bromide into the ionization chamber, however, and on examining the rise in the ionization curve that corresponded to the critical ionization wave-length of bromine we found that the breadth of this rise almost exactly equalled the geometrical estimate of the slit and source correction. In the above mentioned experiment, for instance, it amounted to 1′ 42″. The difference between this and 1′ 40″ is much less than experimental errors; so that we cannot say that a critical ionization has any perceptible breadth of its own. Curve C in Fig. 3 represents a rise in an ionization curve at the critical ionization wave-length.

We have made experiments on the critical ionization of bromine during each one of our measurements of h. We have, therefore, data from which, incidentally, we can calculate a very accurate value of its wave-length. In the four experiments recorded in our tables we have obtained values of the corresponding glancing angle that do not differ from their mean value, 8° 43′ 7″, by more than 5″. This gives for the wave-length of the critical ionization of bromine $\lambda = .91796 \times 10^{-8}$cm.

Table 4 column 2 contains the values of the glancing angle, θ, as measured by curves similar to A and B (Fig. 3).

TABLE 4

Glancing Angles and Radiation Constant

Date	θ (Uncorrected)	$\Delta\theta$	θ(Corrected)	V sin θ	h$\times 10^{27}$
March 15	4° 46′ 43″	47″	4° 47′ 30″	2039.2	6.5539
March 21	4° 46′ 53″	43″	4° 47′ 36″	2039.9	6.5561
March 30	4° 46′ 53″	51″	4° 47′ 44″	2041.0	6.5594
April 5	4° 46′ 48″	45″	4° 47′ 33″	2039.6	6.5552

Column 3 contains the correction, $\Delta\theta$, to be added to θ, determined in each case as described above, and column 4 contains the corrected values of θ. In determining the value of h we calculated the wave length, λ, by means of the equation $\lambda = 2d\sin\theta$, with $2d = 6.056 \times 10^{-8}$cm, then the frequency, ν, from the equation $\lambda\nu = c = 2.9986 \times 10^{10}$ and finally h from the equation $Ve = h\nu$, with $e = 4.774 \times 10^{-10}$.

Evidently what we measure is the product $V\sin\theta$, which, by the way, is not an absolute constant of calcite, for it depends upon the distance between the crystal planes, and this in turn depends upon the temperature, etc. Column 5 in Table 4 contains the values of $V\sin\theta$, the average of which is $V\sin\theta = 2039.9$ at about 20° C, with an estimated error of about $\pm .9$. The grating constant 2d has been calculated for 20° C. The product $V\lambda = 12{,}354$ is independent of the temperature.

The values of h appear in the 6th column of Table 4. The mean value is $h \times 10^{27} = 6.556$ with an estimated error of precision (i.e. without taking account of errors in d, c and e) of $\pm .003$. If we introduce the errors in d, c and e, the probable error in h comes out about $\pm .009$.

This value of h agrees with that previously published by Blake and Duane, (l.c.) but is a fraction of one per cent larger than that recently obtained by E. Wagner in a careful series of measurements.

In the experiments described above the x-rays left the target in a direction at right angles to the line of motion of the cathode particles. An interesting question has been raised recently as to whether the limit of the continuous spectrum would be altered, if the rays came off at some other angle.[5] To test this point with the accurate method of measuring the voltage which we now have, we made a series of experiments with an ordinary Coolidge tube (tungsten target) placed so that the x-rays that passed through the spectrometer's slit left the target at an acute angle of about 45°. If there is any appreciable Doppler effect, this should decrease the value of h as computed by the above equa-

[5] See E. Wagner (l.c.) and G. Zecher, Ann. d. Physik, Sept., 1920, p 28.

tions. The results of these measurement appear in Table 5. As the x-ray tube had no thin mica window, the accuracy is probably somewhat less than before. The value of $V\sin\theta$, however, does

TABLE 5

Glancing Angles and Radiation Constant

Date	θ(Uncorrected)	$\Delta\theta$	θ(Corrected)	$V\sin\theta$	$h\times10^{27}$
April 6	4° 46′ 35″	1′	4° 47′ 35″	2039.8	6.556
April 6	4° 46′ 33″	1′	4° 47′ 33″	2039.6	6.555
April 12	4° 46′ 53″	48″	4° 47′ 41″	2040.7	6.558
April 27	4° 46′ 43″	47″	4° 47′ 30″	2039.3	6.554

Difference of Potential Applied to Tube = 24413 volts.

not differ from that for rays at right angles to the cathode stream. There does not appear to be a Doppler effect for the short wave length limit of the spectrum that amounts to as much as one part in two thousand. This agrees with Wagner's results.

HARVARD UNIVERSITY,
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